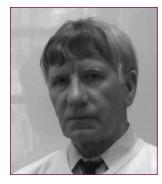# Mathematical notation in Probability & Statistics

IMS Fellow Jordan Stoyanov (Newcastle University, UK, Bulgarian Academy of Sciences and Shandong University, China) wants your involvement in finding agreement on a set of rules for distinct and well-recognizable notations and abbreviations for writings in Probability and Statistics:

When writing papers, books, reports and reviews, we follow good traditions and specific requirements. While most of the notations and abbreviations used in the literature are more or less natural, clear, appropriate and distinct, there are some that are distracting, misleading or just plain tasteless—original and exotic inventions, far from any common sense.

I have discussed these issues with several well-experienced colleagues over the last two decades. Many of them agreed that the current notations and abbreviations in our area, even those that have been used for years, need to be improved and unified. Somebody has to summarize existing good practice and start both speaking publicly about this issue and making specific initial suggestions. Hopefully, this may attract the attention of experienced colleagues and involve them to come up with new and valuable suggestions. Since as they say, "two heads are better than one," if even 1% of the 4,000 or so members of the IMS are willing to take part in this project, the result will be great.

Based on my 50 years' experience as a reader, author, teacher, reviewer and editor, I have found good reasons to write this well-intended Essay. The first version of it was distributed recently to colleagues worldwide, and many authors and a couple of editors expressed strong support. Some authors wrote to tell me how happy they are about switching to a sans serif font for probability, expectation and variance. Yes, there are skeptics. However, it is never too late to take steps in the right direction and enjoy the good outcome.

It is natural to distribute this Essay among a wide audience hoping an appropriate platform for discussions to be found. Thus, if there is a common wish among the scientific community, a compromise can, and should, be found—followed by well selected suggestions to be recommended for implementation. Here is the question: Who will do this, and when?

I am kindly appealing to my fellow probabilists and statisticians, to all mathematicians, to publishers, editors, authors, reviewers, with reference to the saying: If not us, then who? If not now, then when?

Three reasonable motivating rules

Rule 1. Use one letter/symbol as the notation for only one item/object.
Rule 2. For each specific group of objects, use the same font.
Rule 3. For different groups of objects use different fonts.

Good choices for fonts

Use sans serif for probability, expectation, variance, covariance; "mathbb" for spaces of reals, integers, complex numbers; "mathcal" for classes, families.

Some terrible examples

Are you comfortable with the following *terrible* examples met in the literature?

Example 1: Let the IID $\{X_n\}$ be ID with DD… (Meaning: IID = "independent and identically distributed", ID = "infinitely divisible", DD = "decreasing dependence".)

Example 2: The probability $\Pr(\xi \leq x)$, its expectation $\mathbb{E}\,\xi$ and variance $\mathrm{var}\,\xi$… (Here "Pr", "E", and "var" are in one group of objects, the same font must be used, say, sans serif.)

Example 3: For a RV $\mathbb{X}$ on the probability space $(\Omega, \mathcal{F}, \mathbb{P})$ with values in $\mathbb{R}$, denote $\mathbb{E}[\mathbb{X}] = \int_\Omega \mathbb{X}(\omega)d\mathbb{P}$; $\mathbb{P}[\mathbb{X} \leq x] = \mathbb{F}(x)$, $x \in \mathbb{R}$. (Here, the same font, mathbb, is used for five different objects. Terrible!)

Natural, simple and clear abbreviations

Some publishers do not allow abbreviations. If allowed, then a good practice is to use the following: i.i.d., r.v., d.f., a.s., m.g.f., p.g.f., ch.f., inf.div. (no space between). The meaning of each is clear at one glance. It is old-fashioned to write RV, PDF, CDF: density is density, why PDF? Why all these capitals? Why CDF, it is cumulative by definition? It is clear enough to write simply "density" and "d.f.".

Mathematics Subject Classification (MSC)

Use the last and universal MSC 2020. Strangely, there are still papers in journals (though not by IMS) referring to the old MSC 2010, and even to MSC 2000.

Traditionally, we first look for works of good content in our specific area. Next, we pay attention to how appropriate, clear and distinct notations are used. Good notations make the works easier to read! This does not apply to people who only write but do not read.

Specific basic notations

Below is a brief list of basic notations (many of which are widely used in Mathematics):

For the standard normal density use φ ("varphi") not ϕ ("phi"). Also use ε, not ϵ.

Use the "mathcal" font for the normal distribution, $\mathcal{N}(0, 1)$ and $\mathcal{N}(\mu, \sigma^2)$, standard and with parameters μ, σ^2.

For spaces and sets, use the "mathbb" font: $\mathbb{R} = (-\infty, +\infty)$, $\mathbb{R}^n$, $\mathbb{R}_+ = [0, \infty)$; $\mathbb{Q}$ for the rationals; $\mathbb{C}$ for the complex plane; $\mathbb{N} = \{1, 2, \ldots\}$ or $\mathbb{Z}_+$, $\mathbb{N}_0 = \{0, 1, 2, \ldots\}$ or $\mathbb{Z}_0$, $\bar{\mathbb{N}} = \{\ldots, -2, -1, 0, 1, 2, \ldots\}$ or $\mathbb{Z}$. But not the "mathbf" font ($\mathbf{R}$, $\mathbf{N}$, etc.), which is reserved for matrices and vectors. See below.

The "mathcal" font is good for families, σ-algebras, sets of functions: $\mathcal{A}$, $\mathcal{B}$, $\mathcal{D}$, $\mathcal{F}$, $\mathcal{C}$, $\mathcal{N}$, $\mathcal{P}$, $\mathcal{M}$, etc.

Use Roman font (e, d, exp, Re, and Im, respectively, for the Napier constant, differentials, exponent, real and imaginary parts of a complex number). Not e, d, exp, and not $\mathcal{R}e$ and $\mathcal{I}m$.

Parentheses/curly brackets: max$\{a, b\}$, min$\{a, b\}$ is better than max(a, b), min(a, b), etc.

For functions (mappings), use $\mapsto$ not $\rightarrow$; the latter is reserved for convergence.

For specified, say s-convergence, use $\xrightarrow{s}$, but not $\rightarrow_s$ or $\rightarrow^s$.

For convergence, instead of $\xrightarrow{n\to\infty}$, it is better to write $n\to\infty$ after.

Keep $\Gamma(\cdot)$ and B$(\cdot, \cdot)$ for the classical Euler's Gamma function and Beta function.

Use $\gamma(a, b)$, or Gamma(a, b), for the gamma distribution with parameters a, b. Not $\Gamma(\alpha, \beta)$.

Use $\beta(a, b)$, or Beta(a, b), for the beta distribution with parameters a, b. Not B(α, β).

Use Exp(λ) for the exponential distribution with parameter λ. But not $\exp_\lambda$.

Use Bin(n, p) for the binomial distribution with parameters n, p.

Use Poi(λ) for the Poisson distribution with parameter λ.

Use Ge(p) or Geo(p) for the geometrical distribution with parameter p.

Keep the "mathbf" font for matrices and vectors: $\mathbf{A}$, $\mathbf{B}$, …, $\mathbf{a}$, $\mathbf{b}$, ….

Use a sans serif font for the following objects/items, as they are all in one group: P; E; V or Var; Cov$(\cdot, \cdot)$; Corr$(\cdot, \cdot)$; for probability, expectation, variance, covariance, correlation. [Note, this font was used by A.N. Kolmogorov in his book 1933 *Grundbegriffe der Wahrscheinlichkeitsrechnung,* and in any of the many translations and editions, including in *Foundations of Probability Theory*, Chelsea Publ. Co., 1956. Although $\mathbb{P}$, $\mathbf{P}$, and Pr are still in use, they should be smoothly switched to sans serif as an excellent option.]

Two symbols: $X \perp Y$ for orthogonal (uncorrelated) r.v.s; $X \perp\!\!\!\perp Y$ for independent r.v.s.

Final words

While the mathematical content is priority number one, no less important are things of a technical nature, like those discussed above. The use of good, consistent notation and abbreviations will lead to an improved face of writing in Mathematics, in particular in Probability and Statistics.

There are masterly written books and masterly designed journals. One of the reasons is that the authors, either by themselves, or following the journal rules, use notions, notations and abbreviations which are distinct and well recognizable. The conclusion is: Let us follow the Masters!

Note that in this Essay, there is no attempt to discuss other important aspects such as recommendations on styles for citations and references. These reserved areas are almost entirely in the hands of publishers and journal editors.

Further Reading

There is a huge number of available sources on the topics discussed above. Below is a small selection of works by Masters:

Karim Abadir, Jan Magnus (2002). Notions in Econometrics: a proposal for a standard. *Econometrics Journal* **6**:1, 76–90.

Luc Devroye (2020). Mathematics fonts. http://luc.devroye.org/math.html

Paul Halmos (1970). How to write Mathematics. *L'Enseignement Mathématique* **16**:1, 123–152.

Donald Knuth (1992). Two notes on notation. *Amer. Math. Monthly* **99**:5, 403–422.

Steven Krantz (2001). *Handbook of Typography for the Mathematical Sciences*. Chapman & Hall/CRC.

Igor Pak (2018). How to write a clear Math paper: Some 21st century tips. *Journal of Humanistic Mathematics* **8**:1, 301–328.

Jean-Pierre Serre. How to write Mathematics badly. https://www.youtube.com/watch?v=ECQyFzzBHlo

Terence Tao (2007). What is good Mathematics. Preprint, Department of Mathematics, UCLA.

Stephen Wolfram (2000). *Mathematical Notation: Past and Future*.

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To share your opinions, ideas and specific suggestions, please email Jordan Stoyanov with your thoughts: stoyanovj@gmail.com.