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A Review of Self-Exciting Spatio-Temporal Point Processes and Their Applications

Alex Reinhart

Abstract. Self-exciting spatio-temporal point process models predict the rate of events as a function of space, time, and the previous history of events. These models naturally capture triggering and clustering behavior, and have been widely used in fields where spatio-temporal clustering of events is observed, such as earthquake modeling, infectious disease, and crime. In the past several decades, advances have been made in estimation, inference, simulation, and diagnostic tools for self-exciting point process models. In this review, I describe the basic theory, survey related estimation and inference techniques from each field, highlight several key applications, and suggest directions for future research.

Key words and phrases: Epidemic-Type Aftershock Sequence, conditional intensity, Hawkes process, stochastic declustering.

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Comment on "A Review of Self-Exciting Spatiotemporal Point Process and Their Applications" by Alex Reinhart

Yosihiko Ogata

Abstract. In my discussion, I would like to comment on our early reactions to Hawkes' enlightening paper on the self-exciting model; further, I would like to comment on developments of the extended models with some applications.

Key words and phrases: Akaike Bayesian Information Criterion (ABIC), Akaike information criterion (AIC), causality analysis, conditional intensity function, empirical Bayesian method, epidemic-type aftershock sequence (ETAS) model, hierarchical space-time ETAS (HIST-ETAS) model, maximum-likelihood method, penalized log-likelihood, statistical seismology, study of earthquake predictability, thinning simulation method.

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Comment on "A Review of Self-Exciting Spatio-Temporal Point Processes and Their Applications" by Alex Reinhart

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Self-Exciting Point Processes: Infections and Implementations

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Abstract. This is a contribution to the discussion of Reinhart's "Review of Self-Exciting Spatio-Temporal Point Processes and Their Applications" [*Statist. Sci.* **33** (2018)], which synthesizes developments from various research fields. Here, I discuss some experiences from modeling the spread of infectious diseases. Furthermore, I try to complement the review with regard to the availability of software for the described models, which I think is essential in "paving the way for new uses."

Key words and phrases: Spatio-temporal modeling, infectious disease epidemiology, statistical software.

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Rejoinder: A Review of Self-Exciting Spatio-Temporal Point Processes and Their Applications

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On the Relationship between the Theory of Cointegration and the Theory of Phase Synchronization

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Abstract. The theory of cointegration has been a leading theory in econometrics with powerful applications to macroeconomics during the last decades. On the other hand, the theory of phase synchronization for weakly coupled complex oscillators has been one of the leading theories in physics for many years with many applications to different areas of science. For example, in neuroscience phase synchronization is regarded as essential for functional coupling of different brain regions. In an abstract sense, both theories describe the dynamic fluctuation around some equilibrium. In this paper, we point out that there exists a very close connection between both theories. Apart from phase jumps, a stochastic version of the Kuramoto equations can be approximated by a cointegrated system of difference equations. As one consequence, the rich theory on statistical inference for cointegrated systems can immediately be applied for statistical inference on phase synchronization based on empirical data. This includes tests for phase synchronization, tests for unidirectional coupling and the identification of the equilibrium from data including phase shifts. We study two examples on a unidirectionally coupled Rössler–Lorenz system and on electrochemical oscillators. The methods from cointegration may also be used to investigate phase synchronization in complex networks. Conversely, there are many interesting results on phase synchronization which may inspire new research on cointegration.

Key words and phrases: Cointegration, phase synchronization, weakly coupled oscillators, driver response relationship, Rössler–Lorenz system.

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Confidentiality and Differential Privacy in the Dissemination of Frequency Tables

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Abstract. For decades, national statistical agencies and other data custodians have been publishing frequency tables based on census, survey and administrative data. In order to protect the confidentiality of individuals represented in the data, tables based on original data are modified before release. Recently, in response to user demand for more flexible and responsive table publication services, frequency table publication schemes have been augmented with on-line table generating servers such as the US Census Bureau FactFinder and the Australian Bureau of Statistics (ABS) TableBuilder. These systems allow users to build their own custom tables, and make use of automated perturbation routines to protect confidentiality. Motivated by the growing popularity of table generating servers, in this paper we study confidentiality protection for perturbed frequency tables, including the trade-off with analytical utility, focusing on a version of the ABS TableBuilder as a concrete example of a data release mechanism, and examining its properties. Confidentiality protection is assessed in terms of the differential privacy standard, and this paper can be used as a practical introduction to differential privacy, to calculations related to its application, to the relationship between confidentiality protection and utility and to confidentiality in general.

Key words and phrases: Differential privacy, statistical disclosure control, contingency tables, utility.

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Piecewise Deterministic Markov Processes for Continuous-Time Monte Carlo

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Abstract. Recently, there have been conceptually new developments in Monte Carlo methods through the introduction of new MCMC and sequential Monte Carlo (SMC) algorithms which are based on continuous-time, rather than discrete-time, Markov processes. This has led to some fundamentally new Monte Carlo algorithms which can be used to sample from, say, a posterior distribution. Interestingly, continuous-time algorithms seem particularly well suited to Bayesian analysis in big-data settings as they need only access a small sub-set of data points at each iteration, and yet are still guaranteed to target the true posterior distribution. Whilst continuous-time MCMC and SMC methods have been developed independently we show here that they are related by the fact that both involve simulating a piecewise deterministic Markov process. Furthermore, we show that the methods developed to date are just specific cases of a potentially much wider class of continuous-time Monte Carlo algorithms. We give an informal introduction to piecewise deterministic Markov processes, covering the aspects relevant to these new Monte Carlo algorithms, with a view to making the development of new continuous-time Monte Carlo more accessible. We focus on how and why sub-sampling ideas can be used with these algorithms, and aim to give insight into how these new algorithms can be implemented, and what are some of the issues that affect their efficiency.

Key words and phrases: Bayesian statistics, big data, Bouncy Particle Sampler, continuous-time importance sampling, control variates, SCALE, Zig-Zag Sampler.

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Fractionally Differenced Gegenbauer Processes with Long Memory: A Review

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Abstract. The main objective of this paper is to review and promote the usefulness of generalized fractionally differenced Gegenbauer processes in time series and econometric research endeavours. In particular, theoretical and computational aspects centered around fractionally differenced Gegenbauer processes with long memory together with a number of interesting and elegant extensions will be discussed. In-depth conceptual developments and large scale simulation study results are presented for clarity and completeness. This survey highlights a number of gaps in the existing literature of this subject area and becomes a valuable reference source for time series practitioners.

Key words and phrases: Gegenbauer process, long memory, heteroskedasticity, fractional difference, volatility, spectral density, stationarity, invertibility.

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A Unified Theory of Confidence Regions and Testing for High-Dimensional Estimating Equations

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Abstract. We propose a new inferential framework for constructing confidence regions and testing hypotheses in statistical models specified by a system of high-dimensional estimating equations. We construct an influence function by projecting the fitted estimating equations to a sparse direction obtained by solving a large-scale linear program. Our main theoretical contribution is to establish a unified Z-estimation theory of confidence regions for high-dimensional problems. Different from existing methods, all of which require the specification of the likelihood or pseudo-likelihood, our framework is likelihood-free. As a result, our approach provides valid inference for a broad class of high-dimensional constrained estimating equation problems, which are not covered by existing methods. Such examples include, noisy compressed sensing, instrumental variable regression, undirected graphical models, discriminant analysis and vector autoregressive models. We present detailed theoretical results for all these examples. Finally, we conduct thorough numerical simulations, and a real dataset analysis to back up the developed theoretical results.

Key words and phrases: Post-regularization inference, estimating equations, confidence regions, hypothesis tests, Dantzig selector, instrumental variables, graphical models, discriminant analysis, vector autoregressive models.

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A Conversation with Tom Louis

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Abstract. Thomas A. Louis received his BA in Mathematics from Dartmouth College in 1966 and his Ph.D. in Mathematical Statistics from Columbia University in 1972. He served as a NIH Postdoctoral Fellow at Imperial College, London, from 1972–1973 and has held faculty positions at Boston University, Harvard School of Public Health, the University of Minnesota and the Bloomberg School of Public Health at Johns Hopkins University. In addition, he served as a Senior Statistical Scientist at the RAND Corporation, and as Associate Director for Research and Methodology and Chief Scientist at the U.S. Census Bureau. Tom has served as President of both the Eastern North American Region of the International Biometric Society and as President of the International Biometric Society. He is a Fellow of the American Statistical Association, the American Association for the Advancement of Science and the Institute of Mathematical Statistics. As of January 2018, Tom is *Emeritus* Professor, Department of Biostatistics, Bloomberg School of Public Health, Johns Hopkins University. In addition to his many statistical accomplishments, Tom is a strong advocate for professional development and a life-long lover of time on the water.

Key words and phrases: Biography, Applied Statistics, Bayesian Statistics.

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A Conversation with Jim Pitman

David Aldous

Abstract. Jim Pitman was born in June 1949, received a Ph.D. in 1974 from the University of Sheffield with advisor Terry Speed, and since 1979 has been in the U.C. Berkeley Statistics department. He is known for research on many topics within probability, in particular for a long-running collaboration with Marc Yor on distributional properties of Brownian motion, and for his influential lecture notes *Combinatorial Stochastic Processes*. The following conversation took place at his home in December 2017 and February 2018.

Key words and phrases: Mathematical probability, Markov chain, Brownian motion, combinatorial stochastic processes.

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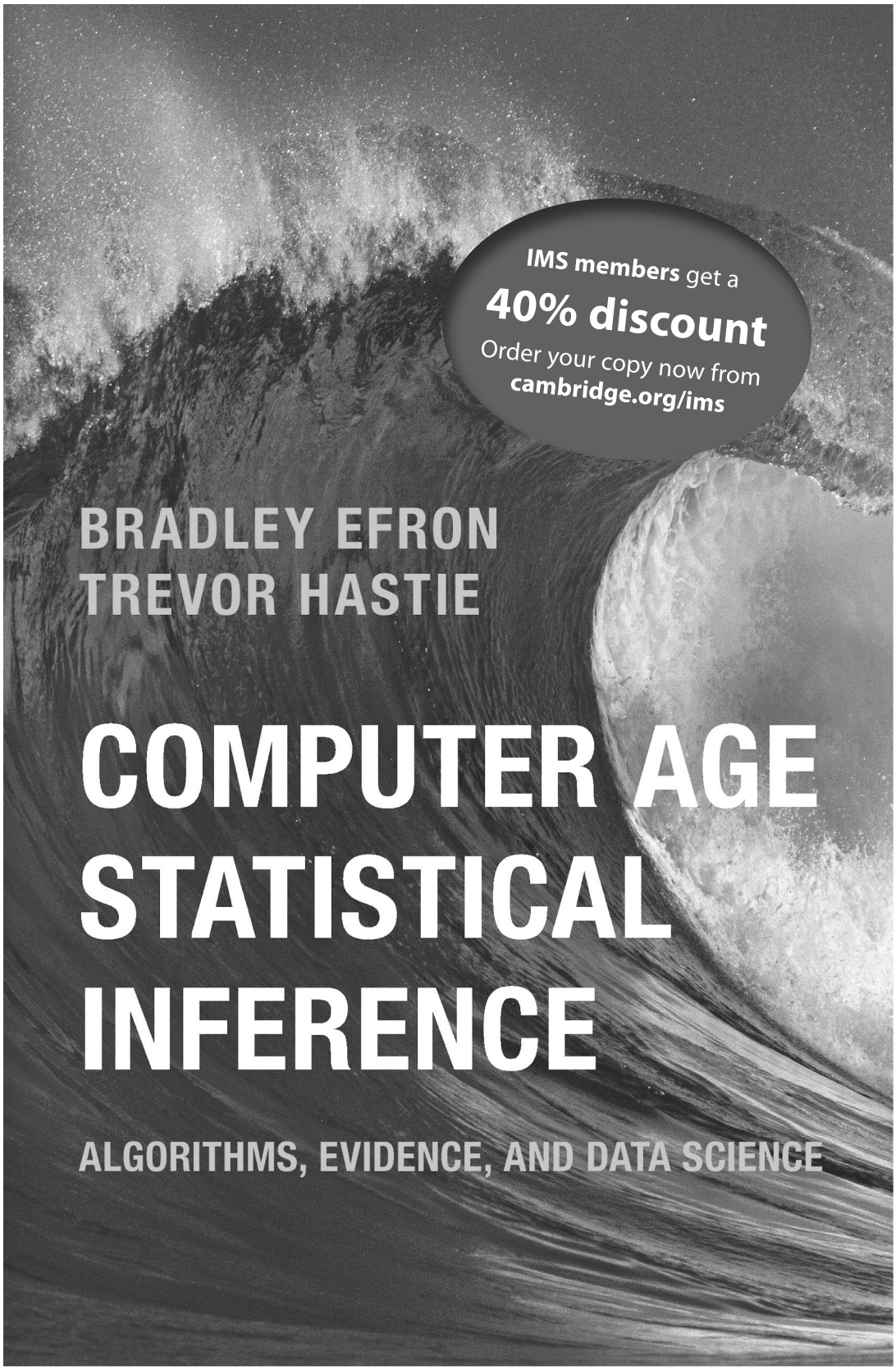
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