Contents

BJPS is an IMS supported journal ISSN 0103-0752 Brazilian Statistical Association

Brazilian Journal of Probability and Statistics

Volume 38 • Number 2 • June 2024

Brazilian Journal of Probability and Statistics

Volume 38 • Number 2 • June 2024

www.imstat.org/bjps

ISSN 0103-0752 (Print) ISSN 2317-6199 (Online), Volume 38, Number 2, June 2024. Published quarterly by the Brazilian Statistical Association.

POSTMASTER:

Send address changes to Brazilian Journal of Probability and Statistics, Institute of Mathematical Statistics, Dues and Subscriptions Office, PO Box 729, Middletown, Maryland 21769, USA.

Brazilian Statistical Association members should send address changes to Rua do Matão, 1010 sala 250A, 05508-090 São Paulo/SP Brazil (address of the BSA office).

Printed in the United States of America

Partial financial support:
CNPq and CAPES (Brazil).

Brazilian Journal of Probability and Statistics
2024, Vol. 38, No. 2, 193–214
https://doi.org/10.1214/24-BJPS600

Different informational characteristics of cubic transmuted distributions

Shital Saha[1,a], **Suchandan Kayal**[1,b,c] **and N. Balakrishnan**[2,d]

[1]*Department of Mathematics, National Institute of Technology Rourkela, Rourkela-769008, Odisha, India,* [a]*shitalmath@gmail.com,* [b]*suchandan.kayal@gmail.com,* [c]*kayals@nitrkl.ac.in*
[2]*Department of Mathematics and Statistics, McMaster University, Hamilton, Ontario L8S 4K1, Canada,* [d]*bala@mcmaster.ca*

Abstract. Cubic transmuted (CT) distributions were introduced recently by Granzotto, Louzada and Balakrishnan (*Journal of Statistical Computation and Simulation* **87** (2017) 2760–2778). In this article, we derive Shannon entropy, Gini's mean difference and Fisher information (matrix) for CT distributions and establish some of their theoretical properties. In addition, we propose cubic transmuted Shannon entropy and cubic transmuted Gini's mean difference. The CT Shannon entropy is expressed in terms of Kullback-Leibler divergences, while the CT Gini's mean difference is shown to be connected with energy distances. We show that the Kullback-Leibler and Chi-square divergences are free of the underlying parent distribution. Finally, we carry out some simulation studies for the proposed information measures from an inferential viewpoint.

References

Ahsan-ul Haq, M., Aldahlan, M. A., Zafar, J., Gómez, H. W., Afify, A. Z. and Mahran, H. A. (2023). A new cubic transmuted power-function distribution: Properties, inference, and applications. *PLoS ONE* **18**, e0281419.

AL-Kadim, K. A. and Mohammed, M. H. (2017). The cubic transmuted Weibull distribution. *Journal of University of Babylon.* **3**, 862–876.

Arnold, B. C., Balakrishnan, N. and Nagaraja, H. N. (1992). *A First Course in Order Statistics*. New York: John Wiley & Sons. MR1178934

Aryal, G. R. and Tsokos, C. P. (2009). On the transmuted extreme value distribution with application. *Nonlinear Analysis: Theory, Methods and Applications.* **71**, 1401–1407. MR2671926 https://doi.org/10.1016/j.na.2009.01.168

Aslam, M., Hussain, Z. and Asghar, Z. (2018). Cubic transmuted-G family of distributions and its properties. *Stochastics and Quality Control* **33**, 103–112. MR3881485 https://doi.org/10.1515/eqc-2017-0027

Balakrishnan, N. (2021). My musings on a pioneering work of Erich Lehmann and its rediscoveries on some families of distributions. *Communications in Statistics—Theory and Methods* **51**, 8066–8073. MR4483940 https://doi.org/10.1080/03610926.2021.1896003

Cramér, H. (1928). On the composition of elementary errors. *Scandinavian Actuarial Journal* **1928**, 13–74.

Di Crescenzo, A. and Longobardi, M. (2006). On weighted residual and past entropies. *Scientiae Mathematicae Japonicae* **64**, 255–266. MR2254144

Dragomir, S., Sunde, J. and Buse, C. (2000). New inequalities for Jeffreys' divergence measure. *Tamsui Oxford University Journal of Mathematical Sciences* **16**, 295–309. MR1833013

Fisher, R. A. (1929). Tests of significance in harmonic analysis. *Proceedings of the Royal Society of London Series A, Containing Papers of a Mathematical and Physical Character* **125**, 54–59.

Granzotto, D., Louzada, F. and Balakrishnan, N. (2017). Cubic rank transmuted distributions: Inferential issues and applications. *Journal of Statistical Computation and Simulation* **87**, 2760–2778. MR3679422 https://doi.org/10.1080/00949655.2017.1344239

Jeffreys, H. (1946). An invariant form for the prior probability in estimation problems. *Proceedings of the Royal Society of London Series A, Mathematical and Physical Sciences* **186**, 453–461. MR0017504 https://doi.org/10.1098/rspa.1946.0056

Key words and phrases. Quadratic transmuted distributions, CT Shannon entropy, CT Gini's mean difference, Fisher information, Kullback-Leibler divergence, Chi-square divergence, cubic transmuted distributions.

Kharazmi, O. and Balakrishnan, N. (2021). Informational properties of transmuted distributions. *Filomat* **35**, 4287–4303. MR4365534

Kullback, S. and Leibler, R. A. (1951). On information and sufficiency. *The Annals of Mathematical Statistics* **22**, 79–86. MR0039968 https://doi.org/10.1214/aoms/1177729694

Lehmann, E. L. (1953). The power of rank tests. *The Annals of Mathematical Statistics* **24**, 23–43. MR0054208 https://doi.org/10.1214/aoms/1177729080

Nielsen, F. and Nock, R. (2013). On the Chi square and higher-order Chi distances for approximating f-divergences. *IEEE Signal Processing Letters* **21**, 10–13.

Rahman, M. M., Al-Zahrani, B. and Shahbaz, M. Q. (2018). A general transmuted family of distributions. *Pakistan Journal of Statistics and Operation Research* **14**, 451–469. MR3814217 https://doi.org/10.18187/pjsor.v14i2.2334

Rahman, M. M., Al-Zahrani, B. and Shahbaz, M. Q. (2019). Cubic transmuted Weibull distribution: Properties and applications. *Annals of Data Science* **6**, 83–102.

Rahman, M. M., Al-Zahrani, B. and Shahbaz, M. Q. (2020). Cubic transmuted Pareto distribution. *Annals of Data Science* **7**, 91–108.

Rényi, A. (1961). On measures of entropy and information. In *Proceedings of the Fourth Berkeley Symposium on Mathematical Statistics and Probability. 1*, 547–561. MR0132570

Shannon, C. E. (1948). A mathematical theory of communication. *The Bell System Technical Journal* **27**, 379–423. MR0026286 https://doi.org/10.1002/j.1538-7305.1948.tb01338.x

Shaw, W. T. and Buckley, I. R. (2009). The alchemy of probability distributions: Beyond Gram-Charlier expansions, and a skew-kurtotic-normal distribution from a rank transmutation map. Preprint. Available at arXiv:0901.0434.

Brazilian Journal of Probability and Statistics
2024, Vol. 38, No. 2, 215–231
https://doi.org/10.1214/24-BJPS601

Flexible conditional density estimation for time series

Gustavo Grivol[1,a], Rafael Izbicki[2,b], Alex A. Okuno[3,c] and Rafael B. Stern[3,d]

[1]*Stern School of Business, New York City, USA* , [a]*ggrivol@gmail.com*
[2]*Department of Statistics, Federal University of Sao Carlos, Sao Carlos, Brazil* , [b]*rafaelizbicki@gmail.com*
[3]*Department of Statistics, University of Sao Paulo, Sao Paulo, Brazil,* [c]*akira.okuno@outlook.com,* [d]*rbstern@gmail.com*

Abstract. Predicting future values of a time series is often insufficient; understanding the complete uncertainty surrounding these predictions is crucial. This necessity has led to the use of conditional density estimators (CDEs). However, many existing CDEs for time series either impose rigid parametric assumptions or exhibit scalability issues when dealing with a high number of covariates. Concurrently, a substantial body of literature has developed around regression methods that offer single-point predictions. In this paper, we introduce a novel approach called FlexCodeTS, which harnesses the power of regression methods to provide enhanced estimates of conditional densities. Through extensive experiments, we demonstrate that FlexCodeTS exhibits strong performance across both real-world and simulated datasets. Furthermore, we establish convergence rates and illustrate how FlexCodeTS can attain rapid convergence by adopting a regression technique that best aligns with the underlying data structure. Finally, we show that FlexCodeTSprovides a straightforward yet accurate measure of variable importance assessment for time series data.

References

Bollerslev, T. (1986). Generalized autoregressive conditional heteroskedasticity. *Journal of Econometrics* **31**, 307–327. MR0853051 https://doi.org/10.1016/0304-4076(86)90063-1

Bollerslev, T. (1987). A conditionally heteroskedastic time series model for speculative prices and rates of return. *Review of Economics and Statistics*, 542–547.

Breiman, L. (2001). Random forests. *Machine Learning* **45**, 5–32. MR3874153

Chen, T., He, T., Benesty, M., Khotilovich, V. and Tang, Y. (2015). Xgboost: Extreme gradient boosting. R package version 0.4-2 1–4.

Chen, X. and White, H. (1999). Improved rates and asymptotic normality for nonparametric neural network estimators. *IEEE Transactions on Information Theory* **45**, 682–691. MR1677026 https://doi.org/10.1109/18.749011

Cho, K., Van Merriënboer, B., Bahdanau, D. and Bengio, Y. (2014). On the properties of neural machine translation: Encoder-decoder approaches. arXiv preprint. Available at arXiv:1409.1259.

Christoffersen, P., Feunou, B., Jeon, Y. and Ornthanalai, C. (2016). Time-varying crash risk: The role of market liquidity.

Dalmasso, N., Pospisil, T., Lee, A. B., Izbicki, R., Freeman, P. E. and Malz, A. I. (2020). Conditional density estimation tools in python and R with applications to photometric redshifts and likelihood-free cosmological inference. *Astronomy and Computing* **30**, 100362.

Dey, B., Zhao, D., Newman, J. A., Andrews, B. H., Izbicki, R. and Lee, A. B. (2022). Calibrated predictive distributions via diagnostics for conditional coverage. arXiv preprint. Available at arXiv:2205.14568.

Engle, R. F. (1982). Autoregressive conditional heteroscedasticity with estimates of the variance of United Kingdom inflation. *Econometrica*, 987–1007. MR0666121 https://doi.org/10.2307/1912773

Fan, J. and Yao, Q. (2008). *Nonlinear Time Series: Nonparametric and Parametric Methods*. New York: Springer. MR2297190 https://doi.org/10.1201/9781420011210

Fernández-Soto, A., Lanzetta, K. M., Chen, H. W., Levine, B. and Yahata, N. (2001). Error analysis of the photometric redshift technique. *Monthly Notices of the Royal Astronomical Society* **330**, 889–894.

Key words and phrases. Nonparametric statistics, conditional density estimation, time series, machine learning.

Filipović, D., Hughston, L. P. and Macrina, A. (2012). Conditional density models for asset pricing. *International Journal of Theoretical and Applied Finance* **15**. MR2902963 https://doi.org/10.1142/S0219024912500021

Freeman, P. E., Izbicki, R. and Lee, A. B. (2017). A unified framework for constructing, tuning and assessing photometric redshift density estimates in a selection bias setting. *Monthly Notices of the Royal Astronomical Society* **468**, 4556–4565.

Gneiting, T. and Katzfuss, M. (2014). Probabilistic forecasting. *Annual Review of Statistics and Its Application* **1**, 125–151.

Hall, P., Racine, J. and Li, Q. (2004). Cross-validation and the estimation of conditional probability densities. *Journal of the American Statistical Association* **99**, 1015–1026. MR2109491 https://doi.org/10.1198/016214504000000548

Hamilton, J. D. (2020). *Time Series Analysis*. New Jersey: Princeton University Press. MR1278033

Hansen, B. E. (1994). Autoregressive conditional density estimation. *International Economic Review*, 705–730.

Härdle, W., Lütkepohl, H. and Chen, R. (1997). A review of nonparametric time series analysis. *International Statistical Review* **65**, 49–72. MR1433592

Hochreiter, S. and Schmidhuber, J. (1997). Long short-term memory. *Neural Computation* **9**, 1735–1780.

Hyndman, R. J. and Yao, Q. (2002). Nonparametric estimation and symmetry tests for conditional density functions. *Journal of Nonparametric Statistics* **14**, 259–278. MR1905751 https://doi.org/10.1080/10485250212374

Izbicki, R., Lee, A. and Schafer, C. (2014). High-dimensional density ratio estimation with extensions to approximate likelihood computation. In *Artificial Intelligence and Statistics*, 420–429. PMLR.

Izbicki, R. and Lee, A. B. (2016). Nonparametric conditional density estimation in a high-dimensional regression setting. *Journal of Computational and Graphical Statistics* **25**, 1297–1316. MR3572041 https://doi.org/10.1080/10618600.2015.1094393

Izbicki, R. and Lee, A. B. (2017). Converting high-dimensional regression to high-dimensional conditional density estimation. *Electronic Journal of Statistics*. MR3679910 https://doi.org/10.1214/17-EJS1302

Izbicki, R., Lee, A. B. and Freeman, P. E. (2017). Photo-z estimation: An example of nonparametric conditional density estimation under selection bias. *Annals of Applied Statistics* **11**, 698–724. MR3693543 https://doi.org/10.1214/16-AOAS1013

Izbicki, R., Lee, A. B. and Pospisil, T. (2020). NNKCDE: Nearest neighbor kernel conditional density estimation. Astrophysics Source Code Library ascl–2005.

Izbicki, R., Shimizu, G. and Stern, R. (2020). Flexible distribution-free conditional predictive bands using density estimators. In *International Conference on Artificial Intelligence and Statistics*, 3068–3077. PMLR. MR4576672

Izbicki, R., Shimizu, G. and Stern, R. B. (2022). CD-split and HPD-split: Efficient conformal regions in high dimensions. *Journal of Machine Learning Research* **23**, 1–32. MR4576672

Jeon, J. and Taylor, J. W. (2012). Using conditional kernel density estimation for wind power density forecasting. *Journal of the American Statistical Association* **107**, 66–79. MR2949342 https://doi.org/10.1080/01621459.2011.643745

Kalda, A. and Siddiqui, S. (2013). Nonparametric conditional density estimation of short-term interest rate movements: Procedures, results and risk management implications. *Applied Financial Economics* **23**, 671–684.

Koenker, R. and Hallock, K. F. (2001). Quantile regression. *The Journal of Economic Perspectives* **15**, 143–156. MR2268657 https://doi.org/10.1017/CBO9780511754098

Lee, A. B. and Izbicki, R. (2016). A spectral series approach to high-dimensional nonparametric regression. *Electronic Journal of Statistics* **10**, 423–463. MR3466189 https://doi.org/10.1214/16-EJS1112

Mallat, S. (1999). *A Wavelet Tour of Signal Processing*. Academic press. MR1614527

Pospisil, T. and Dalmasso, N. (2019). tpospisi/NNKCDE 0.3. https://doi.org/10.5281/zenodo.3364858

Schmidt, S., Malz, A., Soo, J., Almosallam, I., Brescia, M., Cavuoti, S., Cohen-Tanugi, J., Connolly, A., DeRose, J., Freeman, P., et al (2020). Evaluation of probabilistic photometric redshift estimation approaches for the Rubin observatory legacy survey of space and time (LSST). *Monthly Notices of the Royal Astronomical Society* **499**, 1587–1606.

Sentana, E. (1995). Quadratic ARCH models. *The Review of Economic Studies* **62**, 639–661.

Takeuchi, I., Le, Q. V., Sears, T. D., Smola, A. J. and Williams, C. (2006). Nonparametric quantile estimation. *Journal of Machine Learning Research* **7**, 1231–1264. MR2274404 https://doi.org/10.1109/IJCNN.2006.1716084

Truong, Y. K. and Stone, C. J. (1992). Nonparametric function estimation involving time series. *The Annals of Statistics*, 77–97. MR1150335 https://doi.org/10.1214/aos/1176348513

Vaswani, A., Shazeer, N., Parmar, N., Uszkoreit, J., Jones, L., Gomez, A. N., Kaiser, Ł. and Polosukhin, I. (2017). Attention is all you need. In *Advances in Neural Information Processing Systems*, 5998–6008.

Wasserman, L. (2006). *All of Nonparametric Statistics*. New York: Springer. MR2172729

Brazilian Journal of Probability and Statistics
2024, Vol. 38, No. 2, 232–252
https://doi.org/10.1214/24-BJPS602

Efficiency study of a robust regression-type estimator for population mean under different ranked set sampling methods with outlier handling

M. K. Pandey[1,a], **G. N. Singh**[1,b] **and A. Bandyopadhyay**[2,c]

[1]*Department of Mathematics and Computing, Indian Institute of Technology (Indian School of Mines), Dhanbad—826004, India,* [a]*maheshbabu3797@gmail.com,* [b]*gnsingh@iitism.ac.in*

[2]*Department of Mathematics, Amity Institute of Applied Sciences, Amity University Kolkata, Kolkata—700156, India,* [c]*arnabbandyopadhyay4@gmail.com*

Abstract. Classical estimators often suffer from inefficiency when data is contaminated with outliers, leading to skewed results. To overcome this issue, we propose robust estimators for estimating the population mean in various ranked set sampling scenarios. Our research introduces an innovative technique that enhances Zaman estimators under simple random sampling (SRS) and extends these advancements to ranked set sampling (RSS) and median-ranked set sampling (MRSS) designs. Real and simulated datasets, incorporating outliers, are employed to compare the performance of the proposed robust estimator using robust regression methods, that is, LAD, LMS, LTS, Huber-M, Hampel-M, Turkey-M, and Huber-MM, against classical and Zaman estimators. The results demonstrate the superior performance of our robust regression-type estimator in effectively handling outliers.

References

Al-Nasser, A. D. (2007). L ranked set sampling: A generalization procedure for robust visual sampling. *Communications in Statistics-Simulation and Computation* **36**, 33–43. MR2370887 https://doi.org/10.1080/03610910601096510

Al-Omari, A. I. (2012). Ratio estimation of the population mean using auxiliary information in simple random sampling and median ranked set sampling. *Statistics & Probability Letters* **82**, 1883–1890. MR2970287 https://doi.org/10.1016/j.spl.2012.07.001

Al-Omari, A. I. and Bouza, C. N. (2014). Review of ranked set sampling: Modifications and applications. *Revista Investigación Operacional* **3**, 215–240. MR3266914

Al-Saleh, M. F. and Al-Kadiri, M. A. (2000). Double-ranked set sampling. *Statistics & Probability Letters* **48**, 205–212. MR1769826 https://doi.org/10.1016/S0167-7152(99)00206-0

Ali, N., Ahmad, I., Hanif, M. and Shahzad, U. (2021). Robust-regression-type estimators for improving mean estimation of sensitive variables by using auxiliary information. *Communications in Statistics-Theory and Methods* **50**, 979–992. MR4206008 https://doi.org/10.1080/03610926.2019.1645857

Atkinson, A. C. and Cheng, T. C. (1999). Computing least trimmed squares regression with the forward search. *Statistics and Computing* **9**, 251–263.

Audu, A., Ishaq, O. O., Singh, R. V. K., Danbaba, A. and Manu, F. (2023). On the study of efficiency of exponential-type estimator of population mean using robust regression methods. *Quality and Reliability Engineering International* **39**, 190–205.

Blanchet, J., Murthy, K. and Si, N. (2022). Confidence regions in Wasserstein distributionally robust estimation. *Biometrika* **109**, 295–315. MR4430959 https://doi.org/10.1093/biomet/asab026

Blazsek, S. and Escribano, A. (2022). Robust estimation and forecasting of climate change using score-driven ice-age models. *Econometrics* **10**, 9.

Collins, J. R. (1976). Robust estimation of a location parameter in the presence of asymmetry. *The Annals of Statistics*, 68–85. MR0400538

Daszykowski, M., Kaczmarek, K., Vander Heyden, Y. and Walczak, B. (2007). Robust statistics in data analysis—a review: Basic concepts. *Chemometrics and Intelligent Laboratory Systems* **85**, 203–219.

Key words and phrases. Survey sampling, regression-type estimators, robust estimation, outliers, Mahalanobis distance, ranked set sampling, median ranked set sampling.

Dodge, Y. (2008). Least absolute deviation regression. In *The Concise Encyclopedia of Statistics*, 299–302. New York: Springer.

Dua, D. and Graff, C. (2019). *UCI Machine Learning Repository. University of California*. Irvine, CA: School of Information and Computer Science.

Francini, S., McRoberts, R. E., D'Amico, G., Coops, N. C., Hermosilla, T., White, J. C., Wulder, M. A., Marchetti, M., Mugnozza, G. S. and Chirici, G. (2022). An open science and open data approach for the statistically robust estimation of forest disturbance areas. *International Journal of Applied Earth Observation and Geoinformation* **106**, 102663.

Gochanour, B., Chen, S. and Beebe, L. (2022). Multiply robust Bayesian procedures for causal inference problems. *Communications in Statistics-Simulation and Computation*, 1–14.

Grisoni, F., Consonni, V., Vighi, M., Villa, S. and Todeschini, R. (2016). Investigating the mechanisms of bioconcentration through QSAR classification trees. *Environment International* **88**, 198–205.

Hampel, F. R. (1974). The influence curve and its role in robust estimation. *Journal of the American Statistical Association* **69**, 383–393. MR0362657

Hanif, M. and Shahzad, U. (2019a). Estimation of population variance using kernel matrix. *Journal of Statistics & Management Systems* **22**, 563–586.

Huber, P. J. (1973). Robust regression: asymptotics, conjectures, and Monte Carlo. *The Annals of Statistics*. MR0356373

Huber, P. J. (1992). Robust estimation of a location parameter. In *Breakthroughs in Statistics: Methodology and Distribution*, 492–518. Berlin: Springer.

Koyuncu, N. (2016). New difference-cum-ratio and exponential type estimators in median ranked set sampling. *Hacettepe Journal of Mathematics and Statistics* **45**, 207–225. MR3524822 https://doi.org/10.15672/hjms.2015509378

Koyuncu, N. (2018). Regression estimators in ranked set, median ranked set and neoteric ranked set sampling. *Pakistan Journal of Statistics and Operation Research*, 89–94. MR3795500 https://doi.org/10.18187/pjsor.v14i1.1825

Koyuncu, N. (2019). A class of estimators in median ranked set sampling. *İstatistikçiler Dergisi: İstatistik ve Aktüerya* **12**, 58–71. MR3795500 https://doi.org/10.18187/pjsor.v14i1.1825

Koyuncu, N. and Al-Omari, A. I. (2021). Generalized robust-regression-type estimators under different ranked set sampling. *Mathematical Sciences* **15**, 29–40. MR4220859 https://doi.org/10.1007/s40096-020-00360-7

Mahalanobis, P. C. (1930). On tests and measures of group divergence: Theoretical formulae. *Journal and Proceedings of the Asiatic Society of Bengal, New Series* **26**, 541–588.

McIntyre, G. A. (1952). A method for unbiased selective sampling, using ranked sets. *Australian Journal of Agricultural Research* **3**, 385–390.

Muttlak, H. A. (1996). Pair rank set sampling. *Biometrical Journal* **38**, 879–885.

Muttlak, H. A. (1997). Median ranked set sampling. *Journal of Applied Statistical Science* **6**, 245–255.

Pandey, M. K., Singh, G. N. and Bandyopadhyay, A. (2024). Supplement to "Efficiency study of a robust regression-type estimator for population mean under different ranked set sampling methods with outlier handling." https://doi.org/10.1214/24-BJPS602SUPP

Ribeiro, T. K. A. and Ferrari, S. L. P. (2023). Robust estimation in beta regression via maximum L q-likelihood. *Statistical Papers* **64**, 321–353. MR4537405 https://doi.org/10.1007/s00362-022-01320-0

Ripley, B. D. (2002). *Modern Applied Statistics with S*. New York: Springer. MR0875224 https://doi.org/10.1002/9780470316726

Rousseeuw, P. J. (1984). Least median of squares regression. *Journal of the American Statistical Association* **79**, 871–880. MR0770281

Shahzad, U., Ahmad, I., Al-Noor, N. H., Hanif, M. and Almanjahie, I. M. (2023b). Robust estimation of the population mean using quantile regression under systematic sampling. *Mathematical Population Studies* **30**, 195–207. MR4615931 https://doi.org/10.1080/08898480.2022.2139072

Shahzad, U., Ahmad, I., Al-Noor, N. H., Iftikhar, S., Abd Ellah, A. H., Benedict, T. J., et al. (2022b). Särndal approach and separate type quantile robust regression type mean estimators for nonsensitive and sensitive variables in stratified random sampling. *Journal of Mathematics* **2022**. MR4522038 https://doi.org/10.1155/2022/1430488

Shahzad, U., Ahmad, I., Almanjahie, I. M. and Al-Omari, A. I. (2022a). Three-fold utilization of supplementary information for mean estimation under median ranked set sampling scheme. *PLoS ONE* **17**, e0276514.

Shahzad, U., Ahmad, I., Alshahrani, F., Almanjahie, I. M. and Iftikhar, S. (2023a). Calibration-based mean estimators under stratified median ranked set sampling. *Mathematics* **11**, 1825.

Shahzad, U., Al-Noor, N. H., Hanif, M. and Sajjad, I. (2021). An exponential family of median based estimators for mean estimation with simple random sampling scheme. *Communications in Statistics-Theory and Methods* **50**, 4890–4899. MR4308716 https://doi.org/10.1080/03610926.2020.1725828

Shahzad, U., Al-Noor, N. H., Hanif, M., Sajjad, I. and Muhammad Anas, M. (2022c). Imputation based mean estimators in case of missing data utilizing robust regression and variance-covariance matrices. *Communications in Statistics-Simulation and Computation* **51**, 4276–4295. MR4469958 https://doi.org/10.1080/03610918.2020.1740266

Shahzad, U., Perri, P. F. and Hanif, M. (2019b). A new class of ratio-type estimators for improving mean estimation of nonsensitive and sensitive variables by using supplementary information. *Communications in Statistics-Simulation and Computation* **48**, 2566–2585. MR4001221 https://doi.org/10.1080/03610918.2018.1458133

Singh, G. N., Bhattacharyya, D. and Bandyopadhyay, A. (2023). Robust estimation strategy for handling outliers. *Communications in Statistics-Theory and Methods*, 1–20.

Sottile, G. and Frumento, P. (2022). Robust estimation and regression with parametric quantile functions. *Computational Statistics & Data Analysis* **171**, 107471. MR4397088 https://doi.org/10.1016/j.csda.2022.107471

Subza, M., Bouz, C. N. and Al-Omar, A. I. (2019). Utilization of different robust regression techniques for estimation of finite population mean in SRSWOR in case of presence of outliers through ratio method of estimation. *Investigación Operacional* **40**. MR4018940

Tukey, J. W. (1977). *Exploratory Data Analysis*. Boston: Addison-Wesley.

Whatley, M. and Canché, M. S. G. (2022). A robust estimation of the relationship between study abroad and academic outcomes among community college students. *Research in Higher Education* **63**, 271–308.

Zaman, T. and Bulut, H. (2019a). Modified ratio estimators using robust regression methods. *Communications in Statistics-Theory and Methods* **48**, 2039–2048. MR3962099 https://doi.org/10.1080/03610926.2018.1441419

Zaman, T. and Bulut, H. (2019b). Modified regression estimators using robust regression methods and covariance matrices in stratified random sampling. *Communications in Statistics-Theory and Methods*, 1–23. MR4107609 https://doi.org/10.1080/03610926.2019.1588324

Zaman, T., Bulut, H. and Yadav, S. K. (2022). Robust ratio-type estimators for finite population mean in simple random sampling: A simulation study. *Concurrency and Computation Practice and Experience* **34**, e7273.

Zaman, T., Sagir, M. and Şahin, M. (2022). A new exponential estimators for analysis of COVID-19 risk. *Concurrency and Computation Practice and Experience* **34**, e6806.

Zamanzade, E. and Al-Omari, A. I. (2016). New ranked set sampling for estimating the population mean and variance. *Hacettepe Journal of Mathematics and Statistics* **45**, 1891–1905. MR3699745

Zhu, H., Zhang, R., Liu, Y. and Ding, H. (2022). Robust estimation for a general functional single index model via quantile regression. *Journal of the Korean Statistical Society* **51**, 1041–1070. MR4516834 https://doi.org/10.1007/s42952-022-00174-4

Brazilian Journal of Probability and Statistics
2024, Vol. 38, No. 2, 253–265
https://doi.org/10.1214/24-BJPS603

On the optimal pairwise group testing algorithm

Ugnė Čižikovienė[a] **and Viktor Skorniakov**[b]

Institute of Applied Mathematics, Faculty of Mathematics and Informatics, Vilnius University, Naugarduko 24, Vilnius LT-03225, Lithuania, [a]*ugne.cizikoviene@mif.stud.vu.lt,* [b]*viktor.skorniakov@mif.vu.lt*

Abstract. Originally suggested for the blood testing problem by Dorfman in 1943, an idea of Group Testing (GT) has found many applications in other fields as well. Among many (binomial) GT procedures introduced since then, in 1990, Yao and Hwang proposed the Pairwise Testing Algorithm (PTA) and demonstrated that PTA is the *unique* optimal nested GT procedure provided the probability of contamination lies in $\left(1-\frac{\sqrt{2}}{2}, \frac{3-\sqrt{5}}{2}\right)$. Despite the fundamental nature of the result, PTA did not receive considerable attention in the literature. In particular, even its basic probabilistic properties remained unexplored. In this paper, we fill the gap. This is achieved through derivation of an explicit form of the moment generating function (MGF) for the total number of tests T_n required by the PTA procedure. By making use of MGF, we obtain first moments of T_n and characterize its asymptotic behaviour by establishing Law of Large Numbers, Central Limit Theorem and Large Deviation Principle.

References

Abrahams, J. (1993). An improved lower bound on the minimum expected number of binomial group tests. *Probability in the Engineering and Informational Sciences* **7**, 121–124.

Abrahams, J. (1994). Huffman-type codes for infinite source distributions. In *Proceedings of IEEE Data Compression Conference (DCC'94)*, 83–89. Los Alamitos: IEEE. MR1332209 https://doi.org/10.1016/0016-0032(94)90099-X

Abrahams, J. (1997). Code and parse trees for lossless source encoding. In *Proceedings. Compression and Complexity of SEQUENCES 1997*, 145–171. (Cat. No. 97TB100171). MR1907512 https://doi.org/10.4310/CIS.2001.v1.n2.a1

Chi, X.-F., Lou, X.-Y., Yang, M. C. and Shu, Q.-Y. (2009). An optimal DNA pooling strategy for progressive fine mapping. *Genetica* **135**, 267–281.

Dembo, A. and Zeitouni, O. (2009). *Large Deviations Techniques and Applications*. *Stochastic Modelling and Applied Probability*. Berlin: Springer. MR2571413 https://doi.org/10.1007/978-3-642-03311-7

Dorfman, R. (1943). The detection of defective members of large populations. *The Annals of Mathematical Statistics* **14**, 436–440.

Du, D.-Z. and Ko, K.-I. (1987). Some completeness results on decision trees and group testing. *SIAM Journal on Algebraic and Discrete Methods* **8**, 762–777. MR0918075 https://doi.org/10.1137/0608062

Ferguson, T. S. and Tatsuoka, C. (2004). An optimal strategy for sequential classification on partially ordered sets. *Statistics & Probability Letters* **68**, 161–168. MR2066170 https://doi.org/10.1016/j.spl.2004.02.007

Golin, M. J. (2001). A combinatorial approach to Golomb forests. *Theoretical Computer Science* **263**, 283–304. MR1846936 https://doi.org/10.1016/S0304-3975(00)00250-4

Golin, M. J. and Ma, K. K. (2004). Algorithms for infinite Huffman-codes. In *Proceedings of the Fifteenth Annual ACM-SIAM Symposium on Discrete Algorithms*, 758–767. MR2290965

Karimian, S. M. and Straatman, A. G. (2005). Benchmarking of a 3D, unstructured, finite volume code for incompressible Navier-Stokes equation on a cluster of distributed-memory computers. In *19th International Symposium on High Performance Computing Systems and Applications (HPCS'05)*, 11–16. Los Alamitos: IEEE.

Li, M. and Xie, M. (2012). Nonparametric and semiparametric regression analysis of group testing samples. *International Journal of Statistics in Medical Research* **1**, 60–72. MR2941271

Malinovsky, Y. (2020). Conjectures on optimal nested generalized group testing algorithm. *Applied Stochastic Models in Business and Industry* **36**, 1029–1036. MR4201130 https://doi.org/10.1002/asmb.2555

Key words and phrases. Group Testing, Pairwise Testing Algorithm.

Malinovsky, Y. and Albert, P. S. (2019). Revisiting nested group testing procedures: New results, comparisons, and robustness. *American Statistician* **73**, 117–125. MR3953624 https://doi.org/10.1080/00031305.2017.1366367

Malinovsky, Y. and Albert, P. S. (2021). Nested group testing procedures for screening. arXiv preprint. Available at arXiv:2102.03652. MR3953624 https://doi.org/10.1080/00031305.2017.1366367

Sobel, M. (1960). Group testing to classify efficiently all defectives in a binomial sample. In *Information and Decision Processes* (R. E. Machol, ed.) 127–161. New York: McGraw Hill. MR0132667

Sobel, M. (1967). Optimal group testing. In *Proceedings of the Colloquium on Information Theory*, 411–488. Organized by the Bolyai Mathematical Society. MR0258192

Tatsuoka, C. and Ferguson, T. (2003). Sequential classification on partially ordered sets. *Journal of the Royal Statistical Society, Series B, Statistical Methodology* **65**, 143–157. MR1959819 https://doi.org/10.1111/1467-9868.00377

Ungar, P. (1960). The cutoff point for group testing. *Communications on Pure and Applied Mathematics* **13**, 49–54. MR0111113 https://doi.org/10.1002/cpa.3160130105

Wikipedia contributors (2022). NP-completeness — Wikipedia, The Free Encyclopedia. Available at https://en.wikipedia.org/w/index.php?title=NP-completeness&oldid=1091328437. Online; accessed 29-June-2022.

Xie, M. (2001). Regression analysis of group testing samples. *Statistics in Medicine* **20**, 1957–1969.

Xie, M., Tatsuoka, K., Sacks, J. and Young, S. S. (2001). Group testing with blockers and synergism. *Journal of the American Statistical Association* **96**, 92–102. MR1973785 https://doi.org/10.1198/016214501750333009

Yao, Y. C. and Hwang, F. K. (1988). A fundamental monotonicity in group testing. *SIAM Journal on Discrete Mathematics* **1**. MR0941355 https://doi.org/10.1137/0401026

Yao, Y. C. and Hwang, F. K. (1990). On optimal nested group testing algorithms. *Journal of Statistical Planning and Inference* **24**, 167–175. MR1046776 https://doi.org/10.1016/0378-3758(90)90039-W

Brazilian Journal of Probability and Statistics
2024, Vol. 38, No. 2, 266–284
https://doi.org/10.1214/24-BJPS604

Statistical design of ARL-unbiased exponential chart with runs and scans rules using repetitive sampling

Tribhuvan Singh[a]

Department of Mathematics, DIT University, Dehradun, India, [a]*tribhuvansingh94@gmail.com*

Abstract. Exponential chart is widely used to monitor the high-yield processes, i.e., where the failure rate is too low. The exponential chart with runs and scans rules (RSR) scheme has been suggested in the literature for improved performance, especially, for small to moderate shifts. A single sample is used in such a control chart to determine whether the process is in-control (IC) or out-of-control (OOC). However, real-world scenarios may not always permit such conclusive decisions with a single sample. Repetitive sampling can be considered an alternative to single sampling when there is a dispute about the process based on a single sample. In this article, we proposed the exponential chart with the RSR scheme using repetitive sampling and evaluated its performance in terms of average run length (ARL). It is found that the proposed chart is ARL-biased. Therefore, we present a method for designing an ARL-unbiased exponential chart with the RSR scheme using repetitive sampling. The OOC ARL for various shift levels has been computed in order to assess the proposed chart's efficiency. The advantages of the suggested monitoring scheme have been examined and compared with both the existing ARL-unbiased exponential chart using repetitive sampling and the RSR scheme using single sampling. Further, the proposed chart has been compared with the existing control chart using single sampling with roughly matched sample size to the IC average sample size (ASS_0) of repetitive sampling and found to be the control chart using repetitive sampling is efficient with smaller ASS values. Finally, in order to illustrate the designed chart an example has been given.

References

Adeoti, O. A. and Olaomi, J. O. (2018). Capability index-based control chart for monitoring process mean using repetitive sampling. *Communications in Statistics—Theory and Methods* **47**, 493–507. MR3765087 https://doi.org/10.1080/03610926.2017.1307401

Adeoti, O. A. and Rao, G. S. (2022). Attribute control chart for Rayleigh distribution using repetitive sampling under truncated life test. *International Journal of Statistics and Probability* **2022**, 1–11. MR4438036 https://doi.org/10.1155/2022/8763091

Ahmad, L., Aslam, M. and Jun, C. H. (2014). Designing of $\bar{X}$-control charts based on process capability index using repetitive sampling. *Transactions of the Institute of Measurement and Control* **36**, 367–374.

Alevizakos, V., Chatterjee, K., Koukouvinos, C. and Lappa, A. (2023). The effect of parameters estimation on the performance of variables control charts under repetitive sampling. *Communications in Statistics—Theory and Methods* **52**, 2379–2401. MR4554642 https://doi.org/10.1080/03610926.2021.1952269

Aslam, M. (2019). Control chart for variance using repetitive sampling under neutrosophic statistical interval system. *IEEE Access* **7**, 25253–25262.

Aslam, M., Azam, M. and Jun, C. H. (2014a). A new exponentially weighted moving average sign chart using repetitive sampling. *Journal of Process Control* **24**, 1149–1153.

Aslam, M., Khan, N., Azam, M. and Jun, C. H. (2014b). Designing of a new monitoring t-chart using repetitive sampling. *Information Sciences* **269**, 210–216.

Aslam, M., Azam, M. and Jun, C. H. (2014c). New attributes and variables control charts under repetitive sampling. *Industrial Engineering and Management Systems* **13**, 101–106.

Key words and phrases. Average run length, average sample size, control chart, repetitive sampling, runs and scans rules.

Aslam, M., Khan, N. and Jun, C. H. (2015). A new S^2-control chart using repetitive sampling. *Journal of Applied Statistics* **42**, 2485–2496. MR3399299 https://doi.org/10.1080/02664763.2015.1043861

Aslam, M., Niaki, S. T. A., Rasool, M. and Fallahnezhad, M. S. (2012). Decision rule of repetitive acceptance sampling plans assuring percentile life. *Scientia Iranica* **19**, 879–884.

Aslam, M., Yen, C. H. and Jun, C. H. (2011). Variable repetitive group sampling plans with process loss consideration. *Journal of Statistical Computation and Simulation* **81**, 1417–1432. MR2851261 https://doi.org/10.1080/00949655.2010.487826

Balamurali, S. and Jun, C. H. (2006). Repetitive group sampling procedure for variables inspection. *Journal of Applied Statistics* **33**, 327–338. MR2226541 https://doi.org/10.1080/02664760500446010

Balamurali, S., Park, H., Jun, C. H., Kim, K. J. and Lee, J. (2005). Designing of variables repetitive group sampling plan involving minimum average sample number. *Communications in Statistics Simulation and Computation* **34**, 799–809. MR2203582 https://doi.org/10.1081/SAC-200068424

Chakraborti, S., Kumar, N., Rakitzis, A. C. and Sparks, R. S. (2023). Time between events monitoring with control charts. *Wiley StatsRef: Statistics Reference Online*, 1–13. https://doi.org/10.1002/9781118445112.stat08409

Chen, P. W. and Cheng, C. S. (2011). On statistical design of the cumulative quantity control chart for monitoring high-yield processes. *Communications in Statistics—Theory and Methods* **40**, 1911–1928. MR2862688 https://doi.org/10.1080/03610920903391329

Cheng, C. S. and Chen, P. W. (2011). An ARL-unbiased design of time-between-events control charts with runs rules. *Journal of Statistical Computation and Simulation* **81**, 857–871. MR2806930 https://doi.org/10.1080/00949650903520944

Jalilibal, Z., Karavigh, M. H. A., Amiri, A. and Khoo, M. B. C. (2023). Run rules schemes for statistical process monitoring: A literature review. *Quality Technology and Quantitative Management* **20**, 21–52.

Kumar, N., Chakraborti, S. and Rakitzis, A. C. (2017). Improved Shewhart-type charts for monitoring times between events. *Journal of Quality Technology* **49**, 278–296.

Kumar, N., Rakitzis, A. C., Chakraborti, S. and Singh, T. (2024). Statistical design of ATS-unbiased charts with runs rules for monitoring exponential time between events. *Communications in Statistics—Theory and Methods* **53**, 815–833. MR4684439 https://doi.org/10.1080/03610926.2022.2092143

Mahmood, Y., Khoo, M. B. C., Teh, S. Y. and Saha, S. (2022). On designing TEWMA-Tukey control charts for normal and non-normal processes using single and repetitive sampling schemes. *Computers & Industrial Engineering* **170**, 108343.

Malela-Majika, J. C. (2023). Nonparametric precedence chart with repetitive sampling. *Stata Journal* **12**, e512. MR4538653 https://doi.org/10.1002/sta4.512

Muñoz, J. J., Aslam, M. and Campuzano, M. J. (2023). Optimized repetitive sampling $\bar{X}$-control chart: Performance evaluation and comparison with Shewhart control chart. *Frontiers in Applied Mathematics and Statistics* **9**, 1285023.

Nelson, L. S. (1994). A control chart for parts-per-million nonconforming items. *Journal of Quality Technology* **26**, 239–240.

Rakitzis, A. C. (2016). Monitoring exponential data using two-sided control charts with runs rules. *Journal of Statistical Computation and Simulation* **86**, 149–159. MR3403629 https://doi.org/10.1080/00949655.2014.998219

Saleh, N. A., Mahmoud, M. A. and Woodall, W. H. (2023). A re-evaluation of repetitive sampling techniques in statistical process monitoring. *Quality Technology and Quantitative Management*. https://doi.org/10.1080/16843703.2023.2246770

Santiago, E. and Smith, J. (2013). Control charts based on the exponential distribution: Adapting runs rules for the t-chart. *Quality Engineering* **25**, 85–96.

Shafqat, A., Huang, Z. and Aslam, M. (2021). Design of $\bar{X}$-control chart based on inverse Rayleigh distribution under repetitive group sampling. *Ain Shams Engineering Journal* **12**, 943–953.

Sherman, R. E. (1965). Design and evaluation of a repetitive group sampling plan. *Technometrics* **7**, 11–21. MR0174157 https://doi.org/10.2307/1266124

Singh, T. and Kumar, N. (2022). Improved phase II exponential chart under repetitive sampling. *International Journal of Agricultural and Statistics Sciences* **18**, 9–19.

Xie, M., Goh, T. N. and Ranjan, P. (2002). Some effective control chart procedures for reliability monitoring. *Reliability Engineering & System Safety* **77**, 143–150.

Xie, Y. J., Tsui, K. L., Xie, M. and Goh, T. N. (2010). Monitoring time-between-events for health management. Paper presented at: Prognostics and System Health Management Conference, 2010. https://doi.org/10.1109/PHM.2010.5413412

Zhang, C. W., Xie, M., Liu, J. Y. and Goh, T. N. (2007). A control chart for the gamma distribution as a model of time between events. *International Journal of Production Research* **45**, 5649–5666.

Brazilian Journal of Probability and Statistics
2024, Vol. 38, No. 2, 285–301
https://doi.org/10.1214/24-BJPS605

Probabilistic cellular automata with Andrei Toom

Peter Gács[a]

Boston University, Boston, Massachusetts, USA, [a]*gacs@bu.edu*

Abstract. Andrei Toom, who died in September 2022, contributed some of the most fundamental results on probabilistic cellular automata. We want to acquaint the reader with these and will also try to give the reader a look at the environment in which they were born. Toom was an original and strong personality, and other aspects of his life (education, literature) will also deserve mention.

References

Berman, P. and Simon, J. (1988). Investigations of fault-tolerant networks of computers. In *Proc. of the 20-th Annual ACM Symp. on the Theory of Computing*, 66–77.

Bramson, M. and Gray, L. F. (1991). A useful renormalization argument. In *Random Walks, Brownian Motion and Interacting Particle Systems* (R. Durrett and H. Kesten, eds.) 113–152. Boston: Birkhäuser. MR1146444

Chassaing, P. and Mairesse, J. (2011). A non-ergodic probabilistic cellular automaton with a unique invariant measure. *Stochastic Processes and Their Applications* **121**, 2474–2487. MR2832410 https://doi.org/10.1016/j.spa.2011.06.009

de Menezes, M. L. and Toom, A. L. (2006). A non-linear eroder in presence of one-sided noise. *Brazilian Journal of Probability and Statistics* **20**, 1–12. MR2340183

Depoorter, J. and Maes, C. (2006). Stavskaya's measure is weakly Gibbsian. *Markov Processes and Related Fields* **12**, 791–804. MR2284403

Dobrushin, R. L., Kryukov, V. I. and Toom, A. L., eds. (1976). *Locally Interacting Systems and Their Application in Biology: Proceedings of a School-Seminar Held in Puschchino*. *Lecture Notes in Mathematics* **653**. Berlin: Springer.

Durand, B., Romashchenko, A. E. and Shen, A. K. (2012). Fixed-point tile sets and their applications. *Journal of Computer and System Sciences* **78**, 731–764. MR2900032 https://doi.org/10.1016/j.jcss.2011.11.001

Gács, P. (1986). Reliable computation with cellular automata. *Journal of Computer and System Sciences* **32**, 15–78. Conference version at STOC' 83. MR0844202 https://doi.org/10.1016/0022-0000(86)90002-4

Gács, P. (1995). A new version of Toom's proof. Technical report, Department of Computer Science, Boston University, TR 95-009, Boston, MA 02215. Available at arXiv:2105.05968.

Gács, P. (2001). Reliable cellular automata with self-organization. *Journal of Statistical Physics* **103**, 45–267. A corrected and strengthened version available at arXiv:math/0003117 [math.PR]. Early extended abstract: in the proceedings of STOC '97. MR1828729 https://doi.org/10.1023/A:1004823720305

Gács, P., Kurdyumov, G. L. and Levin, L. A. (1978). One-dimensional homogenuous media dissolving finite islands. *Problems of Information Transmission* **14**, 223–226. Translation of the Russian version whose page numbers are 92–96. MR0538903

Gács, P. and Reif, J. (1988). A simple three-dimensional real-time reliable cellular array. *Journal of Computer and System Sciences* **36**, 125–147. Short version in STOC '85. MR0950429 https://doi.org/10.1016/0022-0000(88)90024-4

Gács, P. and Törmä, I. (2022). Stable multi-level monotonic eroders. *Theory of Computing Systems* **66**, 322–353. MR4372254 https://doi.org/10.1007/s00224-021-10061-w

Galperin, G. A. (1976). One-dimensional networks of automata with monotonic local interaction. *Problemy Peredachi Informatsii* **12**, 74–87. Translation from Russian in Problems of Information Transmission. MR0490557

Gray, L. F. (1982). The positive rates problem for attractive nearest neighbor spin systems on $\mathbb{Z}$. *Zeitschrift für Wahrscheinlichkeitstheorie und Verwandte Gebiete* **61**, 389–404. MR0679684 https://doi.org/10.1007/BF00539839

Gray, L. F. (1987). The behavior of processes with statistical mechanical properties. In *Percolation Theory and Ergodic Theory of Infinite Particle Systems* (H. Kesten, ed.) 131–167. Berlin: Springer. MR0894546 https://doi.org/10.1007/978-1-4613-8734-3_9

Key words and phrases. Probabilistic cellular automata.

Gray, L. F. (1999). Toom's stability theorem in continuous time. In *Perplexing Problems in Probability* (H. Kesten, M. Bramson and R. Durrett, eds.) 331–353. Boston: Birkhäuser. MR1703139

Hammersley, J. M. (1967). Harnesses. In *Proceedings of the Fifth Berkeley Symposium on Mathematical Statistics and Probability III*, 89–117. Berkeley: University of California Press. MR0224144

Jahnel, B. and Külske, C. (2015). A class of non-ergodic probabilistic cellular automata with unique invariant measure and quasi-periodic orbit. *Stochastic Processes and Their Applications* **125**, 2427–2450. MR3322870 https://doi.org/10.1016/j.spa.2015.01.006

Kozlov, O. and Vasiliev, N. (1980). Reversible Markov chains with local interaction. In *Multicomponent Random Systems* (R. L. Dobrushin and Y. G. Sinai, eds.), *Advances in Probability and Related Topics* **6**, 451–469. New York: Marcel Dekker Inc. Contributing editor D. Griffeath, series editor P. Ney. Translated from Russian. MR0599544

Kurdyumov, G. L. (1978). An example of a nonergodic homogenous one-dimensional random medium with positive transition probabilities. *Soviet Mathematics Doklady* **19**, 211–214. MR0490601

Kurdyumov, G. L. (1980). An algorithm-theoretic method for the study of uniform random networks. In *Multicomponent Random Systems* (R. L. Dobrushin and Y. G. Sinai, eds.), *Advances in Probability and Related Topics* **6**, 471–503. New York: Marcel Dekker Inc. Contributing editor D. Griffeath, series editor P. Ney. Translated from Russian. MR0599545

Lebowitz, J. L., Maes, C. and Speer, E. R. (1990). Statistical mechanics of stochastic cellular automata. *Journal of Statistical Physics* **59**, 117–170. MR1049965 https://doi.org/10.1007/BF01015566

Liggett, T. M. (1985). *Interacting Particle Systems*. *Grundlehren der mathematischen Wissenschaften* **276**. New York: Springer. Reprinted with new postface in 2005. MR0776231 https://doi.org/10.1007/978-1-4613-8542-4

Malyshev, V. A. (1998). Random graphs and grammars on graphs. *Discrete Mathematics* **10**, 30–44. In Russian. MR1673154 https://doi.org/10.1515/dma.1998.8.3.247

Mityushin, L. G. (1970). Nonergodicity of uniform threshold networks for small self-excitation. *Problemy Peredači Informacii* **6**, 99–103. In Russian. MR0309670

Mountford, T. S. (1995). A coupling of infinite particle systems. *Journal of Mathematics of Kyoto University* **35**, 43–52. MR1317272 https://doi.org/10.1215/kjm/1250518839

Park, K. (1996). Ergodicity and mixing rate of one-dimensional cellular automata. PhD thesis, Boston University. Boston, MA 02215. Available at https://cs-web.bu.edu/faculty/gacs/papers/1996-015-park-phdthesis.pdf. MR2694652

Pippenger, N. (1988). Developments in "The synthesis of reliable organisms from unreliable components". In *The Legacy of John Von Neumann* (J. Glimm, J. Impagliazzo and I. Singer, eds.), *Proceedings of Symposia in Pure Mathematics* **50**, 311–324. Hempstead, NY: American Mathematical Society. MR1067764 https://doi.org/10.1090/pspum/050/1067764

Rocha, A., Simas, A. B. and Toom, A. L. (2011). Substitution operators. *Journal of Statistical Physics* **143**, 585–618. MR2799955 https://doi.org/10.1007/s10955-011-0192-0

Santana, L. H. D., Ramos, A. D. and Toom, A. (2015). Eroders on a plane with three states at a point. Part I: Deterministic. *Journal of Statistical Physics* **159**, 1175–1195. MR3345415 https://doi.org/10.1007/s10955-015-1226-9

Shnirman, M. G. (1968). To the question of the ergodicity of some Markov chain with an inifinite number of states. *Problemy Kibernetiki* **20**, 115–122. In Russian. MR0298777

Stavskaya, O. N. and Piatetski-Shapiro, I. (1968). Homogenous networks of spontaneously active elements. *Problemy Kibernetiki* **20**, 91–106. In Russian. MR0286511

Toom, A., Ramos, A. D., Rocha, A. and Simas, A. B. (2011). Random processes with variable length. In 28° *Colóquio Brasilieiro de Matematica*, 1–112. MR2815671

Toom, A. L. (1963). The complexity of a scheme of functional elements realizing the multiplication of integers. *Soviet Mathematics Doklady* **3**, 714–716. Translation from Russian. MR0156494

Toom, A. L. (1968). On a family of uniform nets of formal neurons. *Soviet Mathematics Doklady* **9**, 1338–1341.

Toom, A. L. (1972). *On Invariant Measures in Nonergodic Random Media, Vol. 2*, 43–51. Moscow University Press. In Russian.

Toom, A. L. (1973). Nonergodic multidimensional systems of automata. *Problems of Information Transmission* **10**, 239–246. Translation from Russian. Pages 70–79 in the original. MR0469584

Toom, A. L. (1976a). Monotone binary cellular automata. *Problems of Information Transmission* **12**, 33–37. Translation from Russian. Pages 48–54 in the original.

Toom, A. L. (1976b). Unstable multicomponent systems. *Problems of Information Transmission*, **12**, 78–84. Pages 220–225 in the translated version. MR0490608

Toom, A. L. (1978a). Monotonic evolutions in real spaces. In *Localy Interacting Systems and Their Applications in Biology* (R. L. Dobrushin, V. I. Kyukov and A. L. Toom, eds.), *Proceedings of a Summer Seminar Held in Pushchino, 1976*, 1–14. Berlin: Springer. Translation from Russian. MR0505431

Toom, A. L. (1978b). Stability in lattice systems with local prohibitions. In *Interacting Markov Processes and Their Applications in Biology* (R. L. Dobrushin, V. I. Kryukov and A. L. Toom, eds.), *Proceedings of a Summer Seminar Held in Pushchino, Moscow Region, May 1978 74–99*, 74–99. Pushchino: Biological research center. Translation from Russian in Sel. Math. Sov., 1985, v. 4, n.1, 73–92. Available at https://cs-web.bu.edu/faculty/gacs/toomandre-com-backup/my-articles/engmat/PROHIB-E.PDF. MR0505439

Toom, A. L. (1980). Stable and attractive trajectories in multicomponent systems. In *Multicomponent Systems* (R. L. Dobrushin, ed.), *Advances in Probability* **6**, 549–575. New York: Dekker. Translation from Russian. MR0599548

Toom, A. L. (1982). Estimates for measures describing the behavior of stochastic systems with local interaction. In *Interacting Markov Processes and Their Applications to the Mathematical Modeling of Biological Systems* (R. L. Dobrushin, V. I. Kryukov and A. L. Toom, eds.), *Proceedings of a Summer School Held in Pushchino, 1981*, 21–33. Pushchino: Biological research center. In Russian. MR0505439

Toom, A. L. (1993). A Russian teacher in America. *The Journal of Mathematical Behavior* **12**, 117–139.

Toom, A. L. (1994a). On critical phenomena in interacting growth systems. Part I: General. *Journal of Statistical Physics* **74**, 91–109. MR1257816 https://doi.org/10.1007/BF02186808

Toom, A. L. (1994b). On critical phenomena in interacting growth systems. Part II: Bounded growth. *Journal of Statistical Physics* **74**, 111–130. MR1257817 https://doi.org/10.1007/BF02186809

Toom, A. L. (1995a). Cellular automata with errors: Problems for students of probability. In *Topics in Contemporary Probability and Its Applications* (J. L. Snell, ed.), *Probability and Stochastic Series* **117**. Boca Raton, FL: CRC Press. MR1410537

Toom, A. L. (1995b). Simple 1-dimensional systems with superexponential relaxation time. *Journal of Statistical Physics* **80**, 545–563. MR1342241 https://doi.org/10.1007/BF02178547

Toom, A. L. (1997). Tails in harnesses. *Journal of Statistical Physics* **88**, 347–364. MR1468388 https://doi.org/10.1007/BF02508475

Toom, A. L. (1998a). On periodic processes of percolation cellular automata. *Resenhas Do Instituto de Matematica E Estatistica Da Universidade de Sao Paulo* **3**, 351–357. MR1633005

Toom, A. L. (1998b). Pseudo-pinning in a growth model. *Markov Processes and Related Fields* **4**, 101–106. MR1677064

Toom, A. L. (2000a). Algorithmic unsolvability of the ergodicity problem for binary cellular automata. *Markov Processes and Related Fields* **6**, 569–577. MR1805094

Toom, A. L. (2000b). Algorithmic unsolvability of the ergodicity problem for locally interacting prcesses with continuous time. *Journal of Statistical Physics* **98**, 495–501. MR1745847 https://doi.org/10.1023/A:1018699527637

Toom, A. L. (2002a). On critical values for some random processes with local interaction in R2. In *And Out of Equilibrium: Probability with a Physics Flavor* (V. Sidoravicius, ed.) 309–319. Boston, MA: Birkhäuser Boston. MR1901960 https://doi.org/10.1007/978-1-4612-0063-5_14

Toom, A. L. (2002b). Particle systems with variable length. *Bulletin Brazilian Mathematical Society* **33**, 419–425. MR1978837 https://doi.org/10.1007/s005740200023

Toom, A. L. (2004). Non-ergodicity in a 1-D particle process with variable length. *Journal of Statistical Physics* **115**, 895–924. MR2054167 https://doi.org/10.1023/B:JOSS.0000022371.44066.f6

Toom, A. L. (2005). Autobiographical notes. Available at http://www.cs.bu.edu/fac/gacs/papers/Pa-longp-eng.pdf.

Toom, A. L. (2007). Every continuous operator has an invariant measure. *Journal of Statistical Physics* **129**, 555–566. MR2351417 https://doi.org/10.1007/s10955-007-9407-9

Toom, A. L. (2009). Ilya Piatetski-Shapiro in 1963–1973: Passion for applied mathematics. Available at https://cs-web.bu.edu/faculty/gacs/toomandre-com-backup/my-articles/engmat/ILYA-ENG.doc.

Toom, A. L. and Mityushin, L. G. (1974). Two results resulting in undecidability for univariate cellular automata. *Problems of Information Transmission*, **12**, 69–75. In the translation journal, pages 135–141. Available at https://cs-web.bu.edu/faculty/gacs/toomandre-com-backup/my-articles/engmat/MITUSH-E.PDF.

Toom, A. L. and Toom, A. (2022). The name of this friendship: Poetic brotherhood. Samizdat. In Russian. Available at https://www.litres.ru/book/andrey-toom-32253688/imya-etoy-druzhby-poeticheskoe-bratstvo-68282777/.

Toom, A. L., Vasiliev, N. B., Stavskaya, O. N., Mityushin, L. G., Kurdyumov, G. L. and Pirogov, S. A. (1990). Discrete local Markov systems. In *Stochastic Cellular Systems: Ergodicity, Memory, Morphogenesis* (R. L. Dobrushin, V. I. Kryukov and A. L. Toom, eds.) 1–182. Manchester: Manchester University Press.

Vasershtein, L. N. (1969). Markov processes over denumerable products of spaces. *Problems of Information Transmission* **5**, 64–72. MR0314115 https://doi.org/10.1016/s0016-0032(33)90010-1

Vasershtein, L. N. and Leontovich, A. M. (1970). On invariant measures of some Markov operators describing a homogenous random medium. *Problems of Information Transmission* **6**, 71–80. In Russian. MR0530400

Vasiliev, N. B. (1970). Correlation equations for the stationary measure of a Markov chain. *Theory of Probability and Its Applications* **15**, 521–525. Translated from Russian. MR0286179

von Neumann, J. (1956). Probabilistic logics and the synthesis of reliable organisms from unreliable components. In *Automata Studies* (C. Shannon and McCarthy, eds.) Princeton, NJ: Princeton University Press. MR0077479

von Neumann, J. (1966). *The Theory of Self-Reproducing Automata*. Urbana: University of Illinois Press.

Brazilian Journal of Probability and Statistics
2024, Vol. 38, No. 2, 302–320
https://doi.org/10.1214/24-BJPS606

Bayesian mixed model for survival data with semicompeting risks based on the Clayton copula

Elizabeth González Patiño[1,a], Gisela Tunes[2,b] and Nelson Ithiro Tanaka[2,c]

[1] *Clinical Trials and Pharmacovigilance Center, Instituto Butantan, São Paulo, Brazil,* [a] *elizabeth.patino@fundacaobutantan.org.br*

[2] *Statistics Department, University of São Paulo, São Paulo, Brazil,* [b] *tunes@ime.usp.br,* [c] *nitanaka@ime.usp.br*

Abstract. Motivated by a chronic kidney disease dataset, we propose a Bayesian model for clustered semicompeting risks data based on Archimedean copulas, allowing for treatment switching. We consider the modeling of both independent and clustered observations. For clustered data, random effects are included to consider the dependence among observations in the same group. For the Clayton copula, we provide theoretical results for the posterior distribution when improper priors are used. A simulation study was conducted to evaluate the performance of the proposed model. Finally, the results of the analysis of chronic kidney disease data are discussed.

References

Brechmann, E. C. and Schepsmeier, U. (2013). Modeling dependence with C- and D-Vine copulas: The R package CDVine. *Journal of Statistical Software* **52**, 1–27. MR3348832 https://doi.org/10.1016/j.jmva.2015.01.001

Casella, G. and George, E. I. (1992). Explaining the Gibbs sampler. *American Statistician* **46**, 167–174. MR1183069 https://doi.org/10.2307/2685208

Chen, M.-H., Shao, Q. and Ibrahim, J. G. (2000). *Monte Carlo Methods in Bayesian Computations*. New York: Springer. MR1742311 https://doi.org/10.1007/978-1-4612-1276-8

Chen, Y. (2012). Maximum likelihood analysis of semicompeting risks data with semiparametric regression models. *Lifetime Data Analysis* **18**, 36–57. MR2878068 https://doi.org/10.1007/s10985-011-9202-4

Demarqui, F. N., Loschi, R. H. and Colosimo, E. A. (2008). Estimating the grid of time-points for the piecewise exponential model. *Lifetime Data Analysis* **14**, 333–356. MR2516849 https://doi.org/10.1007/s10985-008-9086-0

Fine, J. P., Jiang, H. and Chappell, R. (2001). On semicompeting risk. *Biometrika* **88**, 907–919. MR1872209 https://doi.org/10.1093/biomet/88.4.907

Geman, S. and Geman, D. (1984). Stochastic relaxation, Gibbs distributions, and the Bayesian restoration of images. *IEEE Transactions on Pattern Analysis and Machine Intelligence* **6**, 721–741.

Goethals, K., Janssen, P. and Duchateau, L. (2008). Frailty models and copulas: Similarities and differences. *Journal of Applied Statistics* **35**, 1071–1079. MR2522127 https://doi.org/10.1080/02664760802271389

Ibrahim, J. G., Chen, M.-H. and Sinha, D. (2001). *Bayesian Survival Analysis*. New York: Springer. MR1876598 https://doi.org/10.1007/978-1-4757-3447-8

Kaplan, E. L. and Meier, P. (1958). Nonparametric estimation from incomplete observations. *Journal of the American Statistical Association* **53**, 457–481. MR0093867

Lakhal, L., Rivest, L. and Abdous, B. (2008). Estimating survival and association in a semicompeting risk model. *Biometrics* **64**, 180–188. MR2422833 https://doi.org/10.1111/j.1541-0420.2007.00872.x

Lee, K. H., Dominici, F., Schrag, D., et al (2016). Hierarchical models for semi-competing risks data with application to quality of end-of-life care for pancreatic cancer. *Journal of the American Statistical Association* **111**, 1075–1095. MR3561930 https://doi.org/10.1080/01621459.2016.1164052

Lee, K. H., Haneuse, S., Schrag, D., et al (2015). Bayesian semiparametric analysis of semicompeting risks data: Investigating hospital readmission after a pancreatic cancer diagnosis. *Journal of the Royal Statistical Society Series C* **64**, 253–273. MR3302299 https://doi.org/10.1111/rssc.12078

Lesaffre, E. and Lawson, A. B. (2012). *Bayesian Biostatistics*. New Delhi: John Wiley & Sons. MR3236827 https://doi.org/10.1002/9781119942412

Key words and phrases. Semicompeting risks, Archimedean copulas, Clayton copula, data augumentation, chronic kidney disease, mixed model.

Neal, R. M. (2003). Slice sampling. *The Annals of Statistics* **31**, 705–767. MR1994729 https://doi.org/10.1214/aos/1056562461

Patiño, E. G., Tunes, G. and Munera, M. I. (2019). Modeling data with semicompeting risks: An application to chronic kidney disease in Colombia. *Revista Colombiana de Estadística* **42**, 35–60. MR3919837 https://doi.org/10.15446/rce.v42n1.68572

Patiño, E. G., Tunes, G. and Tanaka, N. I. (2024). Supplement to "Bayesian mixed model for survival data with semicompeting risks based on the Clayton copula." https://doi.org/10.1214/24-BJPS606SUPP

Peng, L. and Fine, J. (2007). Regression modeling of semicompeting risk data. *Biometrics* **63**, 96–108. MR2345579 https://doi.org/10.1111/j.1541-0420.2006.00621.x

R Core Team (2021). *R: A Language and Environment for Statistical Computing*. Vienna: R Foundation for Statistical Computing.

Sahu, S. K. and Dey, D. K. (2000). A comparation of frailty and other models for bivariate survival data. *Lifetime Data Analysis* **6**, 207–228.

Spiegelhalter, D. J., Best, N. G., Carlin, B. P., et al (2002). Bayesian measures of model complexity and fit. *Journal of the Royal Statistical Society, Series B* **64**, 583–639. MR1979380 https://doi.org/10.1111/1467-9868.00353

Tanner, T. A. and Wong, W. H. (1987). The calculation of posterior distributions by data augmentation. *Journal of the American Statistical Association* **82**, 528–549. MR0898357

Xu, J., Kalbfleisch, J. and Tai, B. (2010). Statistical analysis of illness-death processes and semicompeting risks data. *Biometrics* **66**, 716–725. MR2758207 https://doi.org/10.1111/j.1541-0420.2009.01340.x

Zeger, S. L. and Karim, M. R. (1991). Generalized linear models with random effects; a Gibbs sampling approach. *Journal of the American Statistical Association* **86**, 79–86. MR1137101

Zeng, D., Chen, Q., Chen, M.-H., Ibrahim, J. G. and AMGEN Research Group (2012). Estimating treatment effects with treatment switching via semicompeting risks models: An application to a colorectal cancer study. *Biometrika* **99**, 167–184. MR2899671 https://doi.org/10.1093/biomet/asr062

Zhang, Y., Chen, M.-H., Ibrahim, J. G., Zeng, D., Chen, Q., Pan, Z. and Xue, X. (2014). Bayesian gamma frailty models for survival data with semi-competing risks and treatment switching. *Lifetime Data Analysis* **20**, 76–105. MR3148185 https://doi.org/10.1007/s10985-013-9254-8

Brazilian Journal of Probability and Statistics
2024, Vol. 38, No. 2, 321–338
https://doi.org/10.1214/24-BJPS607

Functional central limit theorem and Marcinkiewicz strong law of large numbers for Hilbert-valued U-statistics of absolutely regular data

Davide Giraudo[a]

Institut de Recherche Mathématique Avancée UMR 7501, Université de Strasbourg and CNRS 7 rue René Descartes 67000 Strasbourg, France, [a]*dgiraudo@unistra.fr*

Abstract. In this paper, we investigate the functional central limit theorem and the Marcinkiewicz strong law of large numbers for U-statistics having absolutely regular data and taking value in a separable Hilbert space. The novelty of our approach consists in using coupling in order to formulate a deviation inequality for original U-statistic, where the upper bound involves the mixing coefficient and the tail of several U-statistics of i.i.d. data. The presented results improve the known results in several directions: the case of metric space valued data is considered as well as that of Hilbert space valued kernels, and the mixing rates are less restrictive in a wide range of settings.

References

Aaronson, J., Burton, R., Dehling, H., Gilat, D., Hill, T. and Weiss, B. (1996). Strong laws for L- and U-statistics. *Transactions of the American Mathematical Society* **348**, 2845–2866. MR1363941 https://doi.org/10.1090/S0002-9947-96-01681-9

Arcones, M. A. (1998). The law of large numbers for U-statistics under absolute regularity. *Electronic Communications in Probability* **3**, 13–19. MR1624866 https://doi.org/10.1214/ECP.v3-988

Berbee, H. C. P. (1979). *Random Walks with Stationary Increments and Renewal Theory*. *Mathematical Centre Tracts* **112**. Amsterdam: Mathematisch Centrum. MR0547109

Borovkova, S., Burton, R. and Dehling, H. (1999). Consistency of the Takens estimator for the correlation dimension. *The Annals of Applied Probability* **9**, 376–390. MR1687339 https://doi.org/10.1214/aoap/1029962747

Borovkova, S., Burton, R. and Dehling, H. (2001). Limit theorems for functionals of mixing processes with applications to U-statistics and dimension estimation. *Transactions of the American Mathematical Society* **353**, 4261–4318. MR1851171 https://doi.org/10.1090/S0002-9947-01-02819-7

Borovskikh, Y. V. (1986). The theory of U-statistics in a Hilbert space. *Akademii Nauk Ukrainskoj SSR Inst. Mat.* Preprint 78. MR0889097

Borovskikh, Y. V. (1988). The law of large numbers for UH-statistics. *Doklady Akademii Nauk Ukrainskoj SSR Seriâ A 7* **6–9**, 85. MR0968922

Bouzebda, S., Nezzal, A. and Zari, T. (2023). Uniform consistency for functional conditional U-statistics using delta-sequences. *Mathematics* **11**. https://doi.org/10.3390/math11010161

Chakraborty, A. and Chaudhuri, P. (2015). A Wilcoxon–Mann–Whitney-type test for infinite-dimensional data. *Biometrika* **102**, 239–246. MR3335109 https://doi.org/10.1093/biomet/asu072

Chakraborty, A. and Chaudhuri, P. (2017). Tests for high-dimensional data based on means, spatial signs and spatial ranks. *The Annals of Statistics* **45**, 771–799. MR3650400 https://doi.org/10.1214/16-AOS1467

Dedecker, J. and Merlevède, F. (2003). The conditional central limit theorem in Hilbert spaces. *Stochastic Processes and Their Applications* **108**, 229–262. MR2019054 https://doi.org/10.1016/j.spa.2003.07.004

Dedecker, J. and Merlevède, F. (2006). Inequalities for partial sums of Hilbert-valued dependent sequences and applications. *Mathematical Methods of Statistics* **15**, 176–206. MR2256474

Dehling, H., Giraudo, D. and Volný, D. (2023). Some remarks on the ergodic theorem for U-statistics. *Comptes Rendus Mathématique* **361**, 1511–1519. MR4683328 https://doi.org/10.5802/crmath.494

Dehling, H., Sharipov, O. S. and Wendler, M. (2015). Bootstrap for dependent Hilbert space-valued random variables with application to von Mises statistics. *Journal of Multivariate Analysis* **133**, 200–215. MR3282026 https://doi.org/10.1016/j.jmva.2014.09.011

Key words and phrases. Functional central limit theorem, Marcinkiewicz strong law of large numbers, U-statistics.

Dehling, H. and Wendler, M. (2010). Central limit theorem and the bootstrap for U-statistics of strongly mixing data. *Journal of Multivariate Analysis* **101**, 126–137. MR2557623 https://doi.org/10.1016/j.jmva.2009.06.002

Dehling, H. G. and Sharipov, O. S. (2009). Marcinkiewicz–Zygmund strong laws for U-statistics of weakly dependent observations. *Statistics & Probability Letters* **79**, 2028–2036. MR2571765 https://doi.org/10.1016/j.spl.2009.06.012

Fort, G., Moulines, E., Priouret, P. and Vandekerkhove, P. (2012). A simple variance inequality for U-statistics of a Markov chain with applications. *Statistics & Probability Letters* **82**, 1193–1201. MR2915087 https://doi.org/10.1016/j.spl.2012.02.001

Giraudo, D. (2021). Limit theorems for U-statistics of Bernoulli data. *Latin American Journal of Probability and Mathematical Statistics* **18**, 793–828. MR4243516 https://doi.org/10.30757/alea.v18-29

Halmos, P. R. (1946). The theory of unbiased estimation. *The Annals of Mathematical Statistics* **17**, 34–43. MR0015746 https://doi.org/10.1214/aoms/1177731020

Hoeffding, W. (1948). A class of statistics with asymptotically normal distribution. *The Annals of Mathematical Statistics* **19**, 293–325. MR0026294 https://doi.org/10.1214/aoms/1177730196

Hu, W. J., Wang, L., Zhang, B. X. and Wang, G. C. (2023). A two sample test based on U-statistic for functional data. *Acta Mathematica Sinica English Series* **39**, 533–552. MR4573388 https://doi.org/10.1007/s10114-023-1608-x

Jadhav, S. and Ma, S. (2021). An association test for functional data based on Kendall's tau. *Journal of Multivariate Analysis* **184**, 104740. MR4233414 https://doi.org/10.1016/j.jmva.2021.104740

Jiang, F., Wang, R. and Shao, X. (2023). Robust inference for change points in high dimension. *Journal of Multivariate Analysis* **193**, 105114. MR4499388 https://doi.org/10.1016/j.jmva.2022.105114

Leucht, A. (2012). Degenerate U- and V-statistics under weak dependence: Asymptotic theory and bootstrap consistency. *Bernoulli* **18**, 552–585. MR2922461 https://doi.org/10.3150/11-BEJ354

Pinelis, I. (1994). Optimum bounds for the distributions of martingales in Banach spaces. *Annals of Probability* **22**, 1679–1706. MR1331198

Puri, M. L. and Sazonov, V. V. (1993). On the central limit theorem in Hilbert space with application to U-statistics. In *Statistical Sciences and Data Analysis (Tokyo, 1991)* 407–413. Utrecht: VSP. MR1337065

Rio, E. (2000). *Théorie Asymptotique des Processus Aléatoires Faiblement Dépendants*. *Mathématiques & Applications (Berlin) [Mathematics & Applications]* **31**. Berlin: Springer. MR2117923

Sharipov, O. S., Tewes, J. and Wendler, M. (2016). Bootstrap for U-statistics: A new approach. *Journal of Nonparametric Statistics* **28**, 576–594. MR3514512 https://doi.org/10.1080/10485252.2016.1190843

van Delft, A. and Blumberg, A. J. (2023). A statistical framework for analyzing shape in a time series of random geometric objects.

Wegner, L. and Wendler, M. (2023). robust change-point detection for functional time series based on U-statistics and dependent wild bootstrap.

Yoshihara, K.-I. (1976). Limiting behavior of U-statistics for stationary, absolutely regular processes. *Zeitschrift für Wahrscheinlichkeitstheorie und Verwandte Gebiete* **35**, 237–252. MR0418179 https://doi.org/10.1007/BF00532676

Zähle, H. (2014). Marcinkiewicz–Zygmund and ordinary strong laws for empirical distribution functions and plug-in estimators. *Statistics* **48**, 951–964. MR3259868 https://doi.org/10.1080/02331888.2013.800075

AIMS AND SCOPE

The Brazilian Journal of Probability and Statistics aims to publish high quality research papers in applied probability, applied statistics, computational statistics, mathematical statistics, probability theory and stochastic processes.

More specifically, the following types of contributions will be considered:

(i) Original articles dealing with methodological developments, comparison of competing techniques or their computational aspects;
(ii) Original articles developing theoretical results;
(iii) Articles that contain novel applications of existing methodologies to practical problems. For these papers the focus is in the importance and originality of the applied problem, as well as, applications of the best available methodologies to solve it.
(iv) Survey articles containing a thorough coverage of topics of broad interest to probability and statistics. The journal will occasionally publish book reviews, invited papers and essays on the teaching of statistics.

GENERAL INFORMATION

Journal Homepage: https://www.imstat.org/journals-and-publications/brazilian-journal-of-probability-and-statistics/

Submissions: Manuscripts for *Brazilian Journal of Probability and Statistics* should be submitted online. Authors may access the Electronic Journals Management System (EJMS) at https://www.e-publications.org/ims/submission/.

Preparation of Manuscripts: https://imstat.org/journals-and-publications/brazilian-journal-of-probability-and-statistics/brazilian-journal-of-probability-and-statistics-manuscript-submission/

Correspondence. Mail concerning membership, subscriptions, nonreceipt claims, copyright permissions, advertising or back issues should be sent to the IMS Dues and Subscription Office, PO Box 729, Middletown, Maryland 21769, USA. Mail concerning submissions or editorial content should be sent to the Editor at nancy@ime.unicamp.br. Mail concerning the production of this journal should be sent to: Geri Mattson at bjps@mattsonpublishing.com.

Individual and Organizational Memberships:
https://www.imstat.org/individual-membership/

Individual and General Subscriptions:
https://imstat.org/journals-and-publications/subscriptions-and-orders/

Electronic Access to IMS Journals:
https://www.imstat.org/journals-and-publications/electronic-access/

The *Brazilian Journal of Probability and Statistics* is an IMS supported journal:
https://www.imstat.org/journals-and-publications/ims-supported-journals/