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A review of dynamic borrowing methods with applications in pharmaceutical research

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Abstract. This non-technical review discusses the use of historical data in the design and analysis of randomized controlled trials using a Bayesian approach. The focus is on comparing the philosophy behind different approaches and practical considerations for their use. The two main approaches, that is, the power prior and the meta-analytic-predictive prior, are illustrated using fictitious and real data sets. Such methods, which are known as dynamic borrowing methods, are becoming increasingly popular in pharmaceutical research because they may imply an important reduction in costs. In some cases, e.g. in pediatric studies, they may be indispensable to address the clinical research question. In addition to the two original approaches, this review also covers various extensions and variations of the methods. The usefulness and acceptance of the approaches by regulatory agencies is also critically evaluated. Finally, references to relevant software are provided.

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Reinsurance premium estimation for heavy-tailed claim amounts

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Abstract. Using a distortion risk premium principle, we consider estimation of the reinsurance premium when claim amounts are heavy-tailed. We propose two methods to estimate the reinsurance premium. The first one is a non-parametric estimator based directly on the empirical distribution, and the second one is a semi-parametric estimator. Under some regularity conditions, asymptotic normalities of the two estimators are established, and an algorithm for calculating confidence bounds is presented. Further, finite sample behaviors of the two estimators are compared by simulation studies.

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Key words and phrases. Distortion risk premium estimation, Gaussian approximation, heavy tail, regular variation.

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Objective Bayesian analysis for the differential entropy of the Gamma distribution

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Abstract. The paper introduces a fully objective Bayesian analysis to obtain the posterior distribution of an entropy measure. Notably, we consider the gamma distribution, which describes many natural phenomena in physics, engineering, and biology. We reparametrize the model in terms of entropy, and different objective priors are derived, such as Jeffreys prior, reference prior, and matching priors. Since the obtained priors are improper, we prove that the obtained posterior distributions are proper and that their respective posterior means are finite. An intensive simulation study is conducted to select the prior that returns better results regarding bias, mean square error, and coverage probabilities. The proposed approach is illustrated in two datasets: the first relates to the Achaemenid dynasty reign period, and the second describes the time to failure of an electronic component in a sugarcane harvest machine.

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Some estimation procedures for Covid-19 suspected persons in a locality using randomized response model

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Abstract. The current work focuses on incorporating Randomized Response Techniques in Adaptive Cluster Sampling scheme for effective quarantining of COVID-19 suspected individuals, given the sensitive nature of the disease and people's tendency to hide their symptoms. Estimators have been proposed for estimating the number of individuals in a population showing symptoms of COVID-19, the number of individuals in a population not wearing a mask and the optimal size of a quarantine cluster. The effectiveness of the proposed sampling strategy has been demonstrated through empirical studies. Based on the encouraging result, the proposed sampling strategy may be recommended to survey statisticians for their use in the battle against COVID-19 or similar contagious diseases.

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Moment conditions for random coefficient AR(∞) under non-negativity assumptions

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Abstract. We consider random coefficient autoregressive models of infinite order (AR(∞)) under the assumption of non-negativity of the coefficients. We develop novel methods yielding sufficient or necessary conditions for finiteness of moments, based on combinatorial expressions of first and second moments. The methods based on first moments recover previous sufficient conditions by (*Stoch. Process. Their Appl.* **118** (2008) 1997–2013) in our setting. The second moment method provides in particular a necessary and sufficient condition for finiteness of second moments which is different, but shown to be equivalent to the classical criterion of (*Random Coefficient Autoregressive Models: An Introduction* (1982) Springer) in the case of AR(p) models with finite order $p < \infty$. We further illustrate our results through two examples.

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Wellposedness of anticipated BSDEs with quadratic growth and unbounded terminal value

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Abstract. In this paper, we investigate a class of anticipated backward stochastic differential equations (ABSDEs) with quadratic growth and unbounded terminal conditions. ABSDEs give us a duality with stochastic optimal control problems with delay. On the other hand, quadratic ABSDEs can be applied to delayed stochastic linear-quadratic control problems. We prove the well-posedness of ABSDEs with quadratic growth and unbounded terminal values. To obtain the existence result, we first prove a priori estimate for the solutions and then use a limit argument. We also derive a comparison theorem using θ-technique, which gives uniqueness of the solution.

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Properties of solution for fully coupled fractional mean-field forward-backward stochastic differential equation

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Abstract. We consider a fully coupled fractional mean-field forward-backward stochastic differential equation (MF-FBSDE) whose coefficients not only depend on the solution triple (X, Y, Z) but also on its distribution. We prove the existence of a unique solution for such MF-FBSDEs. In addition, we also prove the weak monotonicity and Lipschitz's continuity and a comparison theorem.

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Efficient and robust estimation of tail parameters for Pareto and exponential models

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Abstract. In this paper, a new efficient and robust estimator of the Pareto tail index is proposed. Although the emphasis is on the Pareto distribution, all results are valid for the estimation of the scale/rate parameter of the two-parameter exponential distribution. The approach is to assume that the observations were generated from the FLLP-contaminated Pareto, that is, a mixture of the Pareto and FLLP distributions. The latter is an original distribution designed specifically to represent any outlier distribution. The parameters are estimated using an iterative process adapted from the expectation-maximization (EM) algorithm to optimize the properties of the estimators in a robustness context. A robust confidence interval for the Pareto tail index is also given. It is shown through different asymptotic results that these estimators reach a breakdown point of 50% with full efficiency. Their simultaneous high efficiency and high robustness are also shown for finite samples in a large Monte Carlo simulation study. Finally, an example with a real dataset of daily crude oil returns is given.

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Trajectory fitting estimation for nonlinear stochastic differential equations with reflection

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Abstract. The present paper deals with the problem of trajectory fitting estimation for nonlinear stochastic differential equations with reflection based on continuous-time observation. Under some regularity conditions, the consistency, the rate of convergence and the asymptotic distributions of the trajectory fitting estimator are discussed by using Skorohod equation, Toeplitz lemma and the strong law of large numbers.

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