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THE CRITICAL DENSITY FOR ACTIVATED RANDOM WALKS IS ALWAYS LESS THAN 1

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Activated Random Walks, on $\mathbb{Z}^d$ for any $d \geq 1$, is an interacting particle system, where particles can be in either of two states: active or frozen. Each active particle performs a continuous-time simple random walk during an exponential time of parameter λ, after which it stays still in the frozen state, until another active particle shares its location, and turns it instantaneously back into activity. This model is known to have a phase transition, and we show that the critical density, controlling the phase transition, is less than one in any dimension and for any value of the sleep rate λ. We provide upper bounds for the critical density in both the small λ and large λ regimes.

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DIFFUSIVE SCALING LIMIT OF THE BUSEMANN PROCESS IN LAST PASSAGE PERCOLATION

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In exponential last passage percolation, we consider the rescaled Busemann process

$$x \mapsto N^{-1/3} B^{\rho}_{0,[xN^{2/3}]e_1}, \quad x \in \mathbb{R},$$

as a process parametrized by the scaled density $\rho = 1/2 + \frac{\mu}{4} N^{-1/3}$, and taking values in $C(\mathbb{R})$. We show that these processes, as $N \to \infty$, have a right continuous left limit scaling limit $G = (G_\mu)_{\mu \in \mathbb{R}}$, parametrized by μ and taking values in $C(\mathbb{R})$. The limiting process G, which can be thought of as the Busemann process under the KPZ scaling, can be described as an ensemble of "sticky" lines of Brownian regularity. We believe G is the universal scaling limit of Busemann processes in the KPZ universality class. Our proof provides insight into this limiting behavior by highlighting a connection between the joint distribution of Busemann functions obtained by Fan and Seppäläinen (in *Probab. Math. Phys.* **1** (2020) 55–100), and a sorting algorithm of random walks introduced by O'Connell and Yor (in *Electron. Commun. Probab.* **7** (2002) 1–12).

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The Annals of Probability
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FRACTIONAL DIFFUSION LIMIT FOR A KINETIC FOKKER–PLANCK EQUATION WITH DIFFUSIVE BOUNDARY CONDITIONS IN THE HALF-LINE

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We consider a particle with position $(X_t)_{t\geq 0}$ living in $\mathbb{R}_+$, whose velocity $(V_t)_{t\geq 0}$ is a positive recurrent diffusion with heavy-tailed invariant distribution when the particle lives in $(0,\infty)$. When it hits the boundary $x=0$, the particle restarts with a random strictly positive velocity. We show that the properly rescaled position process converges weakly to a stable process reflected on its infimum. From a P.D.E. point of view, the time-marginals of $(X_t, V_t)_{t\geq 0}$ solve a kinetic Fokker–Planck equation on $(0,\infty)\times\mathbb{R}_+\times\mathbb{R}$ with diffusive boundary conditions. Properly rescaled, the space-marginal converges to the solution of some fractional heat equation on $(0,\infty)\times\mathbb{R}_+$.

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DYNAMICAL LOOP EQUATION

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We introduce dynamical versions of loop (or Dyson–Schwinger) equations for large families of two–dimensional interacting particle systems, including Dyson Brownian motion, Nonintersecting Bernoulli/Poisson random walks, β–corners processes, uniform and Jack-deformed measures on Gelfand–Tsetlin patterns, Macdonald processes, and (q,κ)-distributions on lozenge tilings. Under technical assumptions we show that the dynamical loop equations lead to Gaussian field type fluctuations.

As an application, we compute the limit shape for (q,κ)–distributions on lozenge tilings and prove that their height fluctuations converge to the Gaussian free field in an appropriate complex structure.

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PATH-BY-PATH REGULARISATION THROUGH MULTIPLICATIVE NOISE IN ROUGH, YOUNG, AND ORDINARY DIFFERENTIAL EQUATIONS

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Differential equations perturbed by multiplicative fractional Brownian motions are considered. Depending on the value of the Hurst parameter H, the resulting equation is pathwise viewed as an ODE, YDE, or RDE. In all three regimes, we show regularisation by noise phenomena by proving the strongest kind of well-posedness with irregular drift: strong existence and path-by-path uniqueness. In the Young and smooth regime $H > 1/2$, the condition on the drift coefficient is optimal in the sense that it agrees with the one known for the additive case. In the rough regime $H \in (1/3, 1/2)$, we assume positive but arbitrarily small drift regularity for strong well-posedness, while for distributional drift we obtain weak existence.

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LYAPUNOV EXPONENTS AND SYNCHRONISATION BY NOISE FOR SYSTEMS OF SPDES

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Quantitative estimates for the top Lyapunov exponents for systems of stochastic reaction-diffusion equations are proven. The treatment includes reaction potentials with degenerate minima. The proof relies on an asymptotic expansion of the invariant measure, with careful control on the resulting error terms. As a consequence of these estimates, synchronisation by noise is deduced for systems of stochastic reaction-diffusion equations for the first time.

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CAPACITY OF THE RANGE OF RANDOM WALK: THE LAW OF THE ITERATED LOGARITHM

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We establish both the lim sup and the lim inf law of the iterated logarithm (LIL) for the capacity of the range of a simple random walk in any dimension $d \geq 3$. While for $d \geq 4$, the order of growth in n of such LIL at dimension d matches that for the volume of the random walk range in dimension $d - 2$, somewhat surprisingly this correspondence breaks down for the capacity of the range at $d = 3$. We further establish such LIL for the Brownian capacity of a *three*-dimensional Brownian sample path and novel, sharp moderate deviations bounds for the capacity of the range of a *four*-dimensional simple random walk.

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A LARGE DEVIATION INEQUALITY FOR THE RANK OF A RANDOM MATRIX

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Let A be an $n \times n$ random matrix with independent identically distributed nonconstant sub-Gaussian entries. Then for any $k \le c\sqrt{n}$,

$$\operatorname{rank}(A) \ge n - k$$

with probability at least $1 - \exp(-c'kn)$.

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