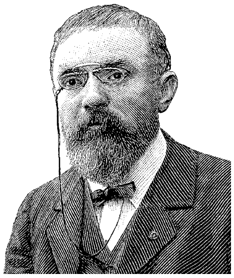

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Deterministic homogenization under optimal moment assumptions for fast-slow systems. Part 1

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Abstract. We consider deterministic homogenization (convergence to a stochastic differential equation) for multiscale systems of the form

$$x_{k+1} = x_k + n^{-1}a_n(x_k, y_k) + n^{-1/2}b_n(x_k, y_k), \quad y_{k+1} = T_n y_k,$$

where the fast dynamics is given by a family T_n of nonuniformly expanding maps. Part 1 builds on our recent work on martingale approximations for families of nonuniformly expanding maps. We prove an iterated weak invariance principle and establish optimal iterated moment bounds for such maps. (The iterated moment bounds are new even for a fixed nonuniformly expanding map T.) The homogenization results are a consequence of this together with parallel developments on rough path theory in Part 2 by Chevyrev, Friz, Korepanov, Melbourne and Zhang.

Résumé. Nous étudions un problème d'homogénéisation déterministe (avec convergence vers une équation différentielle stochastique) pour un système multi-échelle de la forme suivante :

$$x_{k+1} = x_k + n^{-1}a_n(x_k, y_k) + n^{-1/2}b_n(x_k, y_k), \quad y_{k+1} = T_n y_k,$$

où la dynamique rapide est donnée par une famille T_n de transformations non uniformément dilatantes. La partie 1 prolonge nos travaux récents sur l'approximation par des martingales pour des familles de transformations non uniformément dilatantes. Nous montrons un principe d'invariance faible itéré, et établissons des bornes optimales sur les moments dans ce cadre (ces bornes sont nouvelles même pour une transformation non uniformément dilatante T fixée). En combinant ceci et des développements parallèles sur la théorie des chemins rugueux par Chevyrev, Friz, Korepanov, Melbourne et Zhang, nous obtenons les résultats d'homogénéisation dans la partie 2.

MSC2020 subject classifications: Primary 37D25; secondary 37A50; 60F17; 60H10

Keywords: Fast-slow systems; Deterministic homogenization; Martingale decompositions; Iterated moment estimates; Nonuniformly expanding maps and flows

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Deterministic homogenization under optimal moment assumptions for fast–slow systems. Part 2

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Abstract. We consider deterministic homogenization for discrete-time fast–slow systems of the form

$$X_{k+1} = X_k + n^{-1} a_n(X_k, Y_k) + n^{-1/2} b_n(X_k, Y_k), \qquad Y_{k+1} = T_n Y_k$$

and give conditions under which the dynamics of the slow equations converge weakly to an Itô diffusion X as $n \to \infty$. The drift and diffusion coefficients of the limiting stochastic differential equation satisfied by X are given explicitly. This extends the results of Kelly–Melbourne (*J. Funct. Anal.* **272** (2017) 4063–4102) from the continuous-time case to the discrete-time case. Moreover, our methods (p-variation rough paths) work under optimal moment assumptions.

Combined with parallel developments on martingale approximations for families of nonuniformly expanding maps in Part 1 by Korepanov, Kosloff and Melbourne, we obtain optimal homogenization results when T_n is such a family of maps.

Résumé. Nous étudions l'homogénéisation déterministe des systèmes lents-rapides en temps discret de la forme suivante

$$X_{k+1} = X_k + n^{-1} a_n(X_k, Y_k) + n^{-1/2} b_n(X_k, Y_k), \qquad Y_{k+1} = T_n Y_k$$

et donnons des conditions sous lesquelles la dynamique des équations lentes converge en loi vers une diffusion d'Itô X quand $n \to \infty$. Nous calculons explicitement la dérive et les coefficients de diffusion de l'équation différentielle stochastique vérifiée par X. Ceci étend les résultats de Kelly–Melbourne (*J. Funct. Anal.* **272** (2017) 4063–4102) du temps continu au temps discret. De plus, notre méthode (chemins rugueux en p-variation) fonctionne sous des conditions de moments optimales.

Nous obtenons aussi des résultats optimaux d'homogénéisation quand T_n est une famille de transformations non uniformément dilatantes. Ces résultats exploitent les développements parallèles dans la partie 1 (par Korepanov, Kosloff et Melbourne) sur l'approximation par des martingales pour ce type de transformations.

MSC2020 subject classifications: Primary 60H10; secondary 37A50

Keywords: Fast-slow systems; Homogenization; p-variation; Rough paths

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Longtime asymptotics of the two-dimensional parabolic Anderson model with white-noise potential

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Abstract. We consider the parabolic Anderson model (PAM) $\partial_t u = \frac{1}{2}\Delta u + \xi u$ in $\mathbb{R}^2$ with a Gaussian (space) white-noise potential ξ. We prove that the almost-sure large-time asymptotic behaviour of the total mass at time t, written $U(t)$, is given by $\log U(t) \sim \chi t \log t$ for $t \to \infty$, with the deterministic constant χ identified in terms of a variational formula. In earlier work of one of the authors this constant was used to describe the asymptotic behaviour $\boldsymbol{\lambda}_1(Q_t) \sim \chi \log t$ of the principal eigenvalue $\boldsymbol{\lambda}_1(Q_t)$ of the Anderson operator with Dirichlet boundary conditions on the box $Q_t = [-\frac{t}{2}, \frac{t}{2}]^2$.

Résumé. Nous considérons le modèle parabolique d'Anderson (PAM) $\partial_t u = \frac{1}{2}\Delta u + \xi u$ dans $\mathbb{R}^2$ avec un potentiel de bruit blanc ξ en espace. Nous prouvons que le comportement asymptotique presque sûr de la masse totale $U(t)$ au temps t est donnée par $\log U(t) \sim \chi t \log t$ pour $t \to \infty$, avec une constante déterministe χ que nous identifions à l'aide d'une formule variationnelle. Cette constante a déjà été utilisée, dans un travail antérieur de l'un des auteurs, pour décrire le comportement asymptotique $\boldsymbol{\lambda}_1(Q_t) \sim \chi \log t$ de la valeur propre principale $\boldsymbol{\lambda}_1(Q_t)$ de l'opérateur d'Anderson muni de conditions aux limites de Dirichlet sur la boîte $Q_t = [-\frac{t}{2}, \frac{t}{2}]^2$.

MSC2020 subject classifications: Primary 60H17; 60H25; 60L40; 82B44; secondary 35J10; 35P15

Keywords: Parabolic Anderson model; Anderson Hamiltonian; White-noise potential; Singular SPDE; Paracontrolled distribution; Regularization in two dimensions; Intermittency; Almost-sure large-time asymptotics; Principal eigenvalue of random Schrödinger operator

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Weyl law for the Anderson Hamiltonian on a two-dimensional manifold

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Abstract. We define the Anderson Hamiltonian H on a two-dimensional manifold using the high order paracontrolled calculus. It is a self-adjoint operator with pure point spectrum. We get lower and upper bounds on its eigenvalues which imply an almost sure Weyl-type law for H.

Résumé. On définit l'hamiltonien d'Anderson H sur une variété de dimension deux à l'aide du calcul paracontrôlé d'ordre supérieur. C'est un opérateur auto-adjoint avec spectre purement discret. On obtient des bornes inférieures et supérieures sur ses valeurs propres qui impliquent une loi de type Weyl presque sûre pour H.

MSC2020 subject classifications: 35J10; 60H25; 58J05

Keywords: Anderson Hamiltonian; Paracontrolled calculus; White noise; Schrödinger operator

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Fluctuations of the overlap at low temperature in the 2-spin spherical SK model

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Abstract. We describe the fluctuations of the overlap between two replicas in the 2-spin spherical SK model about its limiting value in the low temperature phase. We show that the fluctuations are of order $N^{-1/3}$ and are given by a simple, explicit function of the eigenvalues of a matrix drawn from the Gaussian Orthogonal Ensemble. We show that this quantity converges and describe its limiting distribution in terms of the Airy_1 random point field (i.e., the joint limit of the extremal eigenvalues of the GOE) from random matrix theory.

Résumé. Nous décrivons les fluctuations du produit scalaire entre deux copies ("replicas") dans le modèle de Sherrington–Kirkpatrick quadratique autour de sa valeur limite dans la phase de basse température. Nous démontrons que la taille de ces fluctuations est d'ordre $N^{-1/3}$ et qu'elles s'expriment comme un function simple des valeurs propres d'une matrice aléatoire tirée du Gaussian Orthogonal Ensemble (GOE). Nous montrons que cette quantité converge en loi et décrivons la loi limite à l'aide d'un champ de points aléatoires Airy_1 de la théorie des matrices aléatoires, c'est-à-dire du champ des valeurs propres extremes du GOE.

MSC2020 subject classifications: 60F05

Keywords: Spin glasses; Tracy–Widom distribution

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Random trigonometric polynomials: Universality and non-universality of the variance for the number of real roots

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Abstract. In this paper, we study the number of real roots of random trigonometric polynomials with iid coefficients. When the coefficients have zero mean, unit variance and some finite high moments, we show that the variance of the number of real roots is asymptotically linear in terms of the expectation; furthermore the multiplicative constant in this linear relationship depends only on the kurtosis of the common distribution of the polynomial's coefficients. This result is in sharp contrast to the classical Kac polynomials whose corresponding variance depends only on the first two moments. Our result is perhaps the first paper to establish the variance for general distribution of the coefficients including discrete ones, for a model of random polynomials outside the family of the Kac polynomials. Our method gives a fine comparison framework throughout Edgeworth expansion, asymptotic Kac–Rice formula and a detailed analysis of characteristic functions.

Résumé. Dans cet article, nous étudions le nombre de zéros réels de polynômes trigonométriques avec coefficients i.i.d. Quand les coefficients sont centrés, réduits, et possèdent des moments finis d'ordre suffisamment élevé, nous montrons que la variance du nombre de zéros est asymptotiquement linéaire en son espérance ; de plus, la constante multiplicative dans cette relation linéaire dépend seulement du kurtosis de la loi commune des coefficients du polynôme. Ce résultat contraste fortement avec les classiques polynômes de Kac pour lesquels la variance ne dépend que des deux premiers moments. Il s'agit probablement du premier résultat sur ce type de questions pour des lois générales des coefficients, y compris des lois discrètes, pour des polynômes qui ne sont pas dans la famille des polynômes de Kac. L'expansion de Edgeworth, la formule asymptotique de Kac–Rice et l'analyse précise des fonctions caractéristiques sont les outils principaux de notre approche.

MSC2020 subject classifications: 60G50; 60F05

Keywords: Edgeworth expansion; Random polynomials; Real roots; Universality

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Momenta spacing distributions in anharmonic oscillators and the higher order finite temperature Airy kernel

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Abstract. We rigorously compute the integrable system for the limiting ($N \to \infty$) distribution function of the extreme momentum of N noninteracting fermions when confined to an anharmonic trap $V(q) = q^{2n}$ for $n \in \mathbb{Z}_{\geq 1}$ at positive temperature. More precisely, the edge momentum statistics in the harmonic trap $n = 1$ are known to obey the weak asymmetric KPZ crossover law which is realized via the finite temperature Airy kernel determinant or equivalently via a Painlevé-II integro-differential transcendent, cf. (*Ann. Inst. Henri Poincaré Probab. Stat.* **56** (2) (2020) 1072–1098; *Comm. Pure Appl. Math.* **64** (4) (2011) 466–537). For general $n \geq 2$, a novel higher order finite temperature Airy kernel has recently emerged in physics literature (*Phys. Rev. Lett.* **121** (3) (2018) 030603) and we show that the corresponding edge law in momentum space is now governed by a distinguished Painlevé-II integro-differential hierarchy. Our analysis is based on operator-valued Riemann–Hilbert techniques which produce a Lax pair for an operator-valued Painlevé-II ODE system that naturally encodes the aforementioned hierarchy. As byproduct, we establish a connection of the integro-differential Painlevé-II hierarchy to a novel integro-differential mKdV hierarchy.

Résumé. Nous calculons rigoureusement, pour $N \to \infty$, le système intégrable associé à la fonction de répartition de l'impulsion maximale d'un système de N fermions libres, lorsque les fermions sont confinés dans un potentiel anharmonique $V(q) = q^{2n}$, avec $n \in \mathbb{Z}_{\geq 1}$ et à température positive. Plus précisément, il est bien connu que, dans le cas du potentiel harmonique, i.e. pour $n = 1$, les statistiques de l'impulsion maximale sont décrites par la loi de transition de KPZ dans le régime faiblement asymétrique, qui s'écrit à l'aide du déterminant du noyau d'Airy à température finie ou, de manière équivalente, via une fonction transcendante du type Painlevé II intégro-différentielle (*Ann. Inst. Henri Poincaré Probab. Stat.* **56** (2) (2020) 1072–1098 ; *Comm. Pure Appl. Math.* **64** (4) (2011) 466–537). Pour le cas général $n \geq 2$, un nouveau noyau d'Airy d'ordre supérieur à température finie est récemment apparu dans la littérature en physique (*Phys. Rev. Lett.* **121** (3) (2018) 030603), et nous montrons que la loi limite au bord, dans l'espace des moments, est décrite par une version intégro-différentielle de la hiérarchie de Painlevé II. Notre analyse est basée sur des problèmes de Riemann–Hilbert à valeurs dans les opérateurs, qui produisent une paire de Lax pour un système d'EDO de Painlevé II à valeurs dans les opérateurs. Ce système encode naturellement la hiérarchie susmentionnée. À partir de cette étude, nous établissons aussi une connexion entre la hiérarchie de Painlevé II intégro-différentielle et une nouvelle hiérarchie de Korteweg–de Vries modifiée intégro-différentielle.

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Short cycles in high genus unicellular maps

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Abstract. We study large uniform random maps with one face whose genus grows linearly with the number of edges, which are a model of discrete hyperbolic geometry. In previous works, several hyperbolic geometric features have been investigated. In the present work, we study the number of short cycles in a uniform unicellular map of high genus, and we show that it converges to a Poisson distribution. As a corollary, we obtain the law of the systole of uniform unicellular maps in high genus. We also obtain the asymptotic distribution of the vertex degrees in such a map.

Résumé. Nous étudions les grandes cartes aléatoires uniformes à une face dont le genre croît linéairement avec le nombre d'arêtes, qui sont un modèle de géométrie hyperbolique discrète. Dans des travaux précédents, plusieurs propriétés géométriques hyperboliques ont été étudiées. Dans le présent travail, nous étudions le nombre de petits cycles dans une carte unicellulaire uniforme de grand genre, et nous montrons qu'il converge vers une distribution de Poisson. En corollaire, nous obtenons la loi de la systole des cartes unicellulaires uniformes de grand genre. Nous obtenons également la distribution asymptotique des degrés des sommets dans une telle carte.

MSC2020 subject classifications: 05C10; 05C80; 60C05; 60D05

Keywords: Combinatorial maps; High genus; Short cycles

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Lower bounds for invariant statistical models with applications to principal component analysis

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Abstract. This paper develops nonasymptotic information inequalities for the estimation of the eigenspaces of a covariance operator. These results generalize previous lower bounds for the spiked covariance model, and they show that recent upper bounds for models with decaying eigenvalues are sharp. The proof relies on lower bound techniques based on group invariance arguments. These techniques can also be applied to a variety of other statistical models.

Résumé. Ce travail établit des inégalités d'information non-asymptotiques dans le cadre de l'estimation des espaces propres d'un opérateur de covariance. Ces résultats généralisent d'une part des minorations antérieures valables pour le modèle de perturbation de la covariance, et montrent d'autre part l'optimalité de majorations récentes établies dans des modèles sous contraintes de décroissance des valeurs propres. La preuve repose sur des nouvelles techniques de minoration basées sur des arguments d'invariance de groupes. Ces techniques peuvent également être appliquées à une variété d'autres modèles statistiques.

MSC2020 subject classifications: Primary 62H25; secondary 62B10; 60B20

Keywords: Covariance operator; Principal components; Lower bounds; Van Trees inequality; Fisher information; Equivariant model; Special orthogonal group; Large deviations

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Exact asymptotics of the stochastic wave equation with time-independent noise

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Abstract. In this article, we study the stochastic wave equation in all dimensions $d \leq 3$, driven by a Gaussian noise $\dot{W}$ which does not depend on time. We assume that either the noise is white, or the covariance function of the noise satisfies a scaling property similar to the Riesz kernel. The solution is interpreted in the Skorohod sense using Malliavin calculus. We obtain the exact asymptotic behaviour of the p-th moment of the solution either when the time is large or when p is large. For the critical case, that is the case when $d = 3$ and the noise is white, we obtain the exact transition time for the second moment to be finite.

Résumé. Dans cet article, nous étudions l'équation des ondes stochastique en dimensions $d \leq 3$, dirigée par un bruit gaussien $\dot{W}$ qui ne dépend pas du temps. On suppose que soit le bruit est blanc, soit la fonction de covariance du bruit satisfait une propriété d'échelle similaire au noyau de Riesz. La solution est interprétée dans le sens de Skorohod en utilisant le calcul de Malliavin. On obtient le comportement asymptotique exact du p-ième moment de la solution soit lorsque le temps est grand, soit lorsque p est grand. Pour le cas critique, i.e. si $d = 3$ et le bruit est blanc, on obtient le temps de transition exact pour que le deuxième moment soit fini.

MSC2020 subject classifications: Primary 60H15; secondary 60H07; 37H15

Keywords: Stochastic partial differential equations; Stochastic wave equation; Malliavin calculus; Lyapunov exponents; Exact moment asymptotics; Moment blowup

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Pairwise near-maximal grand coupling of Brownian motions

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Abstract. The well-known reflection coupling gives a maximal coupling of two one-dimensional Brownian motions with different starting points. Nevertheless, the reflection coupling does not generalize to more than two Brownian motions. In this paper, we construct a coupling of all Brownian motions with all possible starting points (i.e., a grand coupling), such that the coupling for any pair of the coupled processes is close to being maximal, that is, the distribution of the coupling time of the pair approaches that of the maximal coupling as the time tends to 0 or ∞, and the coupling time of the pair is always within a multiplicative factor $2e^2$ from the maximal one. We also show that a grand coupling that is pairwise exactly maximal does not exist.

Résumé. Le couplage par réflexion de deux processus browniens est bien connu et ça donne un couplage maximal si on part de deux points distincts. Néanmoins ce couplage ne s'étend pas à plusieurs points de départ. Ici, nous construisons un couplage de tous les mouvements browniens à partir de tous les points de départ (c'est-à-dire un grand couplage) où le couplage de chaque paire est presque maximal. Plus précisément, la distribution du temps de couplage de chaque paire approche celui du couplage maximal à temps zéro et à l'infini, et le temps du couplage de chaque paire est au plus $2e^2$ fois le temps du couplage maximal. Nous démontrons également qu'un grand couplage où le couplage de chaque paire de points de départ est maximal n'existe pas.

MSC2020 subject classifications: 60J65

Keywords: Brownian motion; Reflection coupling; Maximal coupling; Grand coupling; Bessel process

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Pathwise vs. path-by-path uniqueness

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Abstract. We construct a series of stochastic differential equations of the form $dX_t = b(t, X_t)\,dt + dB_t$ which exhibit *nonuniqueness* in the *path-by-path* sense while having a unique adapted solution in the sense of stochastic processes, i.e. *pathwise* uniqueness holds.

Résumé. Nous construisons une série d'équations différentielles stochastiques de la forme $dX_t = b(t, X_t)\,dt + dB_t$ qui présentent *non-unicité* au sens *chemin-par-chemin* tout en ayant une solution adaptée unique au sens des processus stochastiques.

MSC2020 subject classifications: 60H10; 60J65; 60G17; 34F05

Keywords: Stochastic differential equation; Regularization by noise; Path-by-path uniqueness

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Some inequalities for reversible Markov chains and branching random walks via spectral optimization

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Abstract. We present results relating mixing times to the intersection time of branching random walk (BRW) in which the logarithm of the expected number of particles grows at rate of the spectral-gap gap. This is a finite state space analog of a critical branching process. Namely, we show that the maximal expected hitting time of a state by such a BRW is up to a universal constant larger than the L_∞ mixing-time, whereas under transitivity the same is true for the intersection time of two independent such BRWs.

Using the same methodology, we show that for a sequence of reversible Markov chains, the L_∞ mixing-times $t_{\mathrm{mix}}^{(\infty)}$ are of smaller order than the maximal hitting times t_{hit} iff the product of the spectral-gap and t_{hit} diverges, by establishing the inequality $t_{\mathrm{mix}}^{(\infty)} \le \frac{1}{\mathrm{gap}} \log(et_{\mathrm{hit}} \cdot \mathrm{gap})$. This resolves a conjecture of Aldous and Fill "Reversible Markov chains and random walks on graphs" Open Problem 14.12 asserting that under transitivity the condition that $t_{\mathrm{hit}} \gg \frac{1}{\mathrm{gap}}$ implies mean-field behavior for the coalescing time of coalescing random walks.

Résumé. Nous présentons des résultats qui relient les temps de mélange au temps d'intersection d'une marche aléatoire branchante (BRW) dans laquelle le logarithme du nombre moyen de particules croît au taux du trou spectral gap. Il s'agit d'un analogue en espace d'état fini d'un processus de branchement critique. Plus précisément, nous montrons que le maximum de l'espérance du temps d'atteinte d'un état par une telle BRW est à une constante universelle près, plus grand que le temps de mélange L_∞, alors que sous transitivité le même résultat est vrai pour le temps d'intersection de deux telles BRW indépendantes.

En utilisant la même méthodologie, nous montrons que pour une suite de chaînes de Markov réversibles, les temps de mélange L_∞ $t_{\mathrm{mix}}^{(\infty)}$ sont d'ordre inférieur aux temps d'atteinte maximaux t_{hit} si et seulement si le produit du trou spectral et de t_{hit} diverge, en établissant l'inégalité $t_{\mathrm{mix}}^{(\infty)} \le \frac{1}{\mathrm{gap}} \log(et_{\mathrm{hit}} \cdot \mathrm{gap})$. Ceci résout une conjecture d'Aldous et Fill "Reversible Markov chains and random walks on graphs" Open Problem 14.12 affirmant que sous transitivité la condition $t_{\mathrm{hit}} \gg \frac{1}{\mathrm{gap}}$ implique un comportement de champ moyen pour le temps de coalescence des marches aléatoires coalescentes.

MSC2020 subject classifications: 60J10

Keywords: Mixing times; Hitting times; Spectral optimization; Vertex-transitive graphs; Intersection times; Branching random walk; Spectral-gap; Coalescing random walk

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Precise large deviation estimates for branching process in random environment

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Abstract. We consider a branching process in random environment $\{Z_n\}_{n\geq 0}$, which is a population growth process where individuals reproduce independently of each other with the reproduction law randomly picked at each generation. We describe precise asymptotics of upper large deviations, i.e. $\mathbb{P}[Z_n > e^{\rho n}]$. Moreover in the subcritical case, under the Cramér condition on the mean of the reproduction law, we investigate large deviation estimates for the first passage times of the branching process in question and of its total population size.

Résumé. Nous considérons un processus de branchement dans un environnement aléatoire $\{Z_n\}_{n\geq 0}$, qui est le processus de croissance d'une population où les individus se reproduisent indépendamment les uns des autres avec la loi de reproduction choisie au hasard à chaque génération. Nous décrivons des asymptotiques précises des grandes déviations supérieures, c'est-à-dire de $\mathbb{P}[Z_n > e^{\rho n}]$. De plus dans le cas sous-critique, sous la condition de Cramér pour la moyenne de la loi de reproduction, nous étudions les estimations des grandes déviations pour le premier temps de passage de ce processus de branchement et de la taille totale de sa population.

MSC2020 subject classifications: 60J80; 60F10

Keywords: Branching process; Random environment; Large deviations; First passage time; Random walk; Central limit theorem; Law of large numbers

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Concentration of Markov chains indexed by trees

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Abstract. An inequality of Marton (*Ann. Probab.* **24** (1996) 857–866) shows that the joint distribution of a Markov chain with uniformly contracting transition kernels exhibits concentration. We generalize this inequality to Markov chains indexed by trees.

Résumé. Une inégalité de Marton (*Ann. Probab.* **24** (1996) 857–866) montre que la loi jointe d'une chaîne de Markov avec des noyaux de transition uniformément contractants présente un phénomène de concentration. Nous généralisons cette inégalité aux chaînes de Markov indexées par des arbres.

MSC2020 subject classifications: 60K35

Keywords: Concentration of measure; Markov chains; Ising model

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SLE scaling limits for a Laplacian random growth model

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Abstract. We consider a model of planar random aggregation from the ALE$(0, \eta)$ family where particles are attached preferentially in areas of low harmonic measure. We find that the model undergoes a phase transition in negative η, where for sufficiently large values the attachment distribution of each particle becomes atomic in the small particle limit, with each particle attaching to one of the two points at the base of the previous particle. This complements the result of Sola, Turner and Viklund for large positive η, where the attachment distribution condenses to a single atom at the tip of the previous particle.

As a result of this condensation of the attachment distributions we deduce that in the limit as the particle size tends to zero the ALE cluster converges to a Schramm–Loewner evolution with parameter $\kappa = 4$ (SLE_4).

We also conjecture that using other particle shapes from a certain family, we have a similar SLE scaling result, and can obtain SLE_κ for any $\kappa \geq 4$.

Résumé. On considère un modèle d'agrégation planaire aléatoire selon la famille ALE$(0, \eta)$, telle que les particules se rattachent préférentiellement à des régions de petites mesures harmoniques. On montre que le modèle exhibe une transition de phases pour des η négatifs, où pour les valeurs suffisamment grandes, la loi du rattachement de chaque particule devient atomique dans la limite des petites particules, et chaque particule est rattachée à une des deux points à la base de la particule précédente. Ceci complète le résultat de Sola, Turner et Viklund pour les grands η positifs, où la loi du rattachement se concentre en l'atome unique à l'extrémité de la particule précédente.

Comme conséquence de cette condensation de la loi du rattachement, nous déduisons que quand la taille de particule tend vers zéro, l'amas ALE converge vers à une évolution de Schramm–Loewner de paramètre $\kappa = 4$ (SLE_4).

Nous conjecturons aussi qu'en utilisant d'autres formes de particules appartenant à une certaine famille, nous obtiendrons un résultat similaire de renormalisation vers un SLE_κ pour tout $\kappa \geq 4$.

MSC2020 subject classifications: 60F99; 60D05; 30C45

Keywords: Scaling limits; Schramm–Loewner evolution; Conformal aggregation; Harmonic measure

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Crossing estimates from metric graph and discrete GFF

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Abstract. We compare level-set percolation for Gaussian free field (GFF) defined on a rectangular subset of $\delta\mathbb{Z}^2$ to level-set percolation for GFF defined on the corresponding metric graph as the mesh size δ goes to 0. In particular, we look at the probability that there is a path that crosses the rectangle in the horizontal direction on which the field is positive. We show this probability is strictly larger in the discrete graph. In the metric graph case, we show that for appropriate boundary conditions the probability that there exists a closed pivotal edge for the horizontal crossing event decays logarithmically in δ. In the discrete graph case, we compute the limit of the probability of a horizontal crossing for appropriate boundary conditions.

Résumé. Nous comparons la percolation des ensembles de niveau d'un champ libre Gaussien (GFF) discret défini sur un sous-ensemble rectangulaire de $\delta\mathbb{Z}^2$ avec la percolation des ensembles de niveau pour le GFF défini sur le graphe métrique correspondant lorsque la taille du maillage passe à zéro. En particulier, on regarde la probabilité qu'il existe un chemin qui traverse le rectangle dans la direction horizontale sur lequel le champ est positif. Nous montrons que cette probabilité est strictement plus grande dans le graphe discret. Dans le cas du graphe métrique, nous montrons que, sous des conditions au bord appropriées, la probabilité qu'il existe une arête pivot fermée pour l'événement de croisement horizontal décroît de manière logarithmique en δ. Dans le cas d'un graphe discret, nous calculons la limite de la probabilité d'un croisement horizontal sous des conditions au bord appropriées.

MSC2020 subject classifications: Primary 60G15; 60G60; secondary 60J67

Keywords: Gaussian free field; Crossing event; Scaling limit; Pivotal edge

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Logarithmic correction to resistance

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Abstract. We study the trace of the incipient infinite oriented branching random walk in $\mathbb{Z}^d \times \mathbb{Z}_+$ when the dimension is $d = 6$. Under suitable moment assumptions, we show that the electrical resistance between the root and level n is $O(n \log^{-\xi} n)$ for a $\xi > 0$ that does not depend on details of the model.

Résumé. Nous étudions la trace de la marche aléatoire branchante critique conditionnée à être infinie sur $\mathbb{Z}^d \times \mathbb{Z}_+$, en dimension $d = 6$. Sous des hypothèses appropriées sur les moments de la marche et de la loi de reproduction, nous démontrons que la résistance électrique entre la racine et le niveau n est d'ordre $O(n \log^{-\xi} n)$ pour un certain $\xi > 0$ qui ne dépend pas des détails du modèle.

MSC2020 subject classifications: Primary 60K50; secondary 60K35; 82C41; 31C20; 60J80

Keywords: Electrical resistance; Branching random walk; Anomalous diffusion

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On the threshold of spread-out contact process percolation

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Abstract. We study the stationary distribution of the (spread-out) d-dimensional contact process from the point of view of site percolation. In this process, vertices of $\mathbb{Z}^d$ can be healthy (state 0) or infected (state 1). With rate one infected sites recover, and with rate λ they transmit the infection to some other vertex chosen uniformly within a ball of radius R. The classical phase transition result for this process states that there is a critical value $\lambda_c(R)$ such that the process has a non-trivial stationary distribution if and only if $\lambda > \lambda_c(R)$. In configurations sampled from this stationary distribution, we study nearest-neighbor site percolation of the set of infected sites; the associated percolation threshold is denoted $\lambda_p(R)$. We prove that $\lambda_p(R)$ converges to $1/(1-p_c)$ as R tends to infinity, where p_c is the threshold for Bernoulli site percolation on $\mathbb{Z}^d$. As a consequence, we prove that $\lambda_p(R) > \lambda_c(R)$ for large enough R, answering an open question of (*Probabilites et Statistiques* **42** (2006) 223–243) in the spread-out case.

Résumé. Nous étudions, du point de vue de la percolation par sites, la distribution stationnaire du processus de contact (avec transmission à longue portée) en dimension d. Dans ce processus, un sommet de $\mathbb{Z}^d$ peut être sain (état 0) ou infecté (état 1). À taux un, les sommets infectés se rétablissent et à taux λ, ils transmettent l'infection à un autre sommet t choisi uniformément dans une boule de rayon R. Le résultat classique de transition de phase pour ce processus indique qu'il existe une valeur critique $\lambda_c(R)$ telle que le processus a une distribution stationnaire non triviale si et seulement si $\lambda > \lambda_c(R)$. Sur les configurations échantillonnées selon distribution stationnaire, nous étudions la percolation par sites et aux plus proches voisins de l'ensemble des sites infectés ; le seuil de percolation associé est noté $\lambda_p(R)$. Nous montrons que $\lambda_p(R)$ converge vers $1/(1-p_c)$ lorsque R tend vers l'infini, où p_c est le seuil de la percolation par sites de Bernoulli sur $\mathbb{Z}^d$. En conséquence, nous prouvons que $\lambda_p(R) > \lambda_c(R)$ pour R assez grand, répondant à une question ouverte de (*Probabilites et Statistiques* **42** (2006) 223–243).

MSC2020 subject classifications: Primary 60K35; secondary 82C22

Keywords: Interacting particle systems; Contact process; Percolation

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Longest increasing paths with Lipschitz constraints

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Abstract. The Hammersley problem asks for the maximal number of points in a monotonous path through a Poisson point process. It is exactly solvable and notoriously known to belong to the KPZ universality class, with a cube-root scaling for the fluctuations. Here we introduce and analyze a variant in which we impose a Lipschitz condition on paths. Thanks to a coupling with the classical Hammersley problem we observe that this variant is also exactly solvable. It allows us to derive first and second order asymptotics. It turns out that the cube-root scaling only holds for certain choices of the Lipschitz constants.

Résumé. Le problème d'Hammersley consiste à étudier nombre maximal de points d'un processus ponctuel de Poisson par lequel un chemin monotone peut passer. Ce problème est exactement soluble et il appartient notoirement à la classe d'universalité KPZ, avec des fluctuations d'ordre la racine cubique du nombre de points pris. Nous introduisons et analysons ici une variante dans laquelle nous imposons une condition de Lipschitz sur les chemins. Grâce à un couplage avec le problème classique d'Hammersley, nous observons que cette variante est également exactement soluble. Ceci nous permet d'en déduire des asymptotiques du premier et du second ordre. Il s'avère que des fluctuations d'ordre racine cubique n'apparaissent que pour certains choix des constantes de Lipschitz.

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