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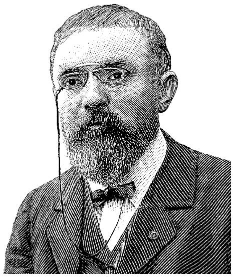

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Multicolour Poisson matching

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Abstract. Consider several independent Poisson point processes on $\mathbb{R}^d$, each with a different colour and perhaps a different intensity, and suppose we are given a set of allowed family types, each of which is a multiset of colours such as red-blue or red-red-green. We study translation-invariant schemes for partitioning the points into families of allowed types. This generalizes the 1-colour and 2-colour matching schemes studied previously (where the sets of allowed family types are the singletons {red-red} and {red-blue} respectively). We characterize when such a scheme exists, as well as the optimal tail behaviour of a typical family diameter. The latter has two different regimes that are analogous to the 1-colour and 2-colour cases, and correspond to the intensity vector lying in the interior and boundary of the existence region respectively.

We also address the effect of requiring the partition to be a deterministic function (i.e. a factor) of the points. Here we find the optimal tail behaviour in dimension 1. There is a further separation into two regimes, governed by algebraic properties of the allowed family types.

Résumé. Nous considérons plusieurs processus ponctuels de Poisson indépendants sur $\mathbb{R}^d$, chacun d'une couleur différente et possiblement d'une intensité différente, et nous supposons donné un ensemble de type de familles, chacun étant un multi-ensemble de couleurs comme rouge-bleu ou rouge-rouge-bleu. Nous étudions les stratégies invariantes par translations pour partitionner les points en familles de types autorisés. Ceci généralise les stratégies d'appariements à une et deux couleurs étudiés précédemment (où l'ensemble des types de familles autorisés sont les singletons rouge-rouge et rouge- bleu respectivement). Nous caractérisons l'existence de telles stratégies, ainsi que le comportement de la queue de distribution du diamètre d'une famille typique. Il existe différents régimes pour cette dernière ; ceux-ci sont analogues aux cas à un et deux couleurs et correspondent au fait que le vecteur d'intensité se trouve à l'intérieur ou à la frontière de la région d'existence, respectivement.

Nous étudions aussi l'effet d'imposer que la partition soit une fonction déterministe (i.e. un facteur) du nuage de points. Dans ce cas, nous trouvons le comportement de la queue de distribution en dimension 1. On observe une nouvelle séparation en deux régimes, déterminés par des propriétés algébriques des types de familles autorisés.

MSC2020 subject classifications: 60D05; 60G55; 05C70

Keywords: Poisson process; Point process; Invariant matching; Invariant partition; Factor map

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Almost square permutations are typically square

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Abstract. A record in a permutation is a maximum or a minimum, from the left or from the right. The entries of a permutation can be partitioned into two types: the ones that are records are called external points, the others are called internal points. Permutations without internal points have been studied under the name of square permutations. Here, we explore permutations with a fixed number of internals points, called almost square permutations. Unlike with square permutations, a precise enumeration for the total number of almost square permutations of size $n+k$ with exactly k internal points is not known. However, using a probabilistic approach, we are able to determine the asymptotic enumeration. This allows us to describe the permuton limit of almost square permutations with k internal points, both when k is fixed and when k tends to infinity along a negligible sequence with respect to the size of the permutation. Finally, we show that our techniques are quite general by studying the set of 321-avoiding permutations of size n with exactly k additional internal points (k fixed). In this case we obtain an interesting asymptotic enumeration in terms of the Brownian excursion area. As a consequence, we show that the points of a uniform permutation in this set concentrate on the diagonal and the fluctuations of these points converge in distribution to a biased Brownian excursion.

Résumé. Dans une permutation, un record est un maximum ou un minimum, en partant de la droite ou de la gauche. Les éléments d'une permutation peuvent être classés en deux types : les records (dits points extérieurs) et les autres (dits points intérieurs). Les permutations sans point intérieur ont été étudiées sous le nom de permutations carrées. Ici, nous explorons les permutations ayant un nombre fini de points intérieurs, dites permutations quasi-carrées. Contrairement aux permutations carrées, l'énumération précise du nombre total de permutations quasi-carrées de taille $n+k$ ayant exactement k points intérieurs n'est pas connue. Cependant, grâce à une approche probabiliste, nous sommes capables d'obtenir une énumération asymptotique. Cela nous permet de décrire la limite du permuton d'une permutation presque-carrée ayant k points internes, pour k fixé ou lorsque k tend vers l'infini selon une suite négligeable par rapport à la taille de la permutation. Enfin, nous montrons que nos techniques sont relativement générales en étudiant l'ensemble des permutations de taille n évitant le motif 321 et ayant exactement k points internes supplémentaires (pour k fixé). Dans ce cas, nous obtenons une énumération asymptotique intéressante en termes d'aire de l'excursion Brownienne. Nous montrons que, par conséquent, les points d'une permutation uniforme dans cet ensemble se concentrent sur la diagonale et que leurs fluctuations convergent en distribution vers une excursion Brownienne biaisée.

MSC2020 subject classifications: 60C05; 05A05; 05A16

Keywords: Permutations; Scaling limits; Permutons; Asymptotic enumeration methods

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Concentration of scalar ergodic diffusions and some statistical implications

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Abstract. We derive uniform concentration inequalities for continuous-time analogues of empirical processes and related stochastic integrals of scalar ergodic diffusion processes. Thereby, we lay the foundation typically required for the study of sup-norm properties of estimation procedures for a large class of diffusion processes. In the classical i.i.d. context, a key device for the statistical sup-norm analysis is provided by Talagrand-type concentration inequalities. Aiming for a parallel substitute in the diffusion framework, we present a systematic, self-contained approach to such uniform concentration inequalities via martingale approximation and moment bounds obtained by the generic chaining method. The developed machinery is of independent probabilistic interest and can serve as a starting point for investigations of other processes such as more general Markov processes, in particular multivariate or discretely observed diffusions. As a first concrete statistical application, we analyse the sup-norm error of estimating the invariant density of an ergodic diffusion via the local time estimator and the classical nonparametric kernel density estimator, respectively.

Résumé. Nous obtenons des inégalités de concentration uniformes pour des analogues en temps continu de processus empiriques ainsi que pour des intégrales stochastiques de diffusions scalaires ergodiques associées. De ce fait, nous posons les bases requises pour l'étude des propriétés en norme-sup de procédures d'estimation d'une vaste classe de processus de diffusion. Dans le contexte i.i.d. classique, un outil-clé pour l'analyse statistique en norme-sup est celui des inégalités de concentration de type Talagrand. Comme substitut à ces outils dans le cadre des diffusions, nous présentons une approche systématique et auto-contenue à de telles inégalités de concentration uniformes à l'aide d'approximation par martingales et de bornes de moments obtenues par la méthode du chaînage générique. Les outils probabilistes ainsi développés sont d'intérêt indépendant et peuvent servir de point de départ à l'étude d'autres processus tels que des processus de Markov plus généraux, en particulier des diffusions multivariées ou observées en des temps discrets. Comme première application statistique concrète, nous analysons l'erreur en norme-sup dans l'estimation de la densité invariante d'une diffusion ergodique, respectivement par l'estimateur du temps local et par l'estimateur classique non-paramétrique à noyau de la densité.

MSC2020 subject classifications: Primary 60J60; 60J55; secondary 62M05; 62G07

Keywords: Ergodic diffusion; Concentration of diffusions; Exponential inequalities; Local time estimator

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Constant payoff in zero-sum stochastic games

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Abstract. In a zero-sum stochastic game, at each stage, two adversary players take decisions and receive a stage payoff determined by them and by a controlled random variable representing the state of nature. The total payoff is the normalized discounted sum of the stage payoffs. In this paper we solve the "constant payoff" conjecture formulated by Sorin, Venel and Vigeral (*Sankhya A* **72** (1) (2010) 237–245): if both players use optimal strategies, then for any $\alpha > 0$, the expected discounted payoff between stage 1 and stage α/λ tends to the limit discounted value of the game, as the discount rate λ goes to 0.

Résumé. Dans un jeu stochastique à somme nulle, à chaque étape, deux joueurs adversaires prennent des décisions et reçoivent un paiement d'étape déterminé par ces décisions, ainsi que par une variable aléatoire contrôlée qui représente l'état de la nature. Le paiement total est la somme escomptée et normalisée des paiements d'étape. Dans cet article, nous résolvons la conjecture du "paiement constant", formulée par Sorin, Venel et Vigeral (*Sankhya A* **72** (1) (2010) 237–245) : si les deux joueurs jouent des stratégies optimales, alors pour tout $\alpha > 0$, l'espérance du paiement escompté entre les étapes 1 et α/λ tend vers la limite de la valeur escomptée du jeu, lorsque le facteur d'escompte λ tend vers 0.

MSC2020 subject classifications: Primary 91A05; 91A15; 91A20; secondary 15B51

Keywords: Zero-sum stochastic games; Constant payoff; Limit value; Puiseux series

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N-Player games and mean-field games with smooth dependence on past absorptions

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Abstract. Mean-field games with absorption is a class of games that has been introduced in (*Ann. Appl. Probab.* **28** (2018) 2188–2242) and that can be viewed as natural limits of symmetric stochastic differential games with a large number of players who, interacting through a mean-field, leave the game as soon as their private states hit some given boundary.

In this paper, we push the study of such games further, extending their scope along two main directions. First, we allow the state dynamics and the costs to have a very general, possibly infinite-dimensional, dependence on the (non-normalized) empirical sub-probability measure of the survivors' states. This includes the particularly relevant case where the mean-field interaction among the players is done through the empirical measure of the survivors together with the fraction of absorbed players over time. Second, the boundedness of coefficients and costs has been considerably relaxed including drift and costs with linear growth in the state variables, hence allowing for more realistic dynamics for players' private states. We prove the existence of solutions of the MFG in strict as well as relaxed feedback form, and we establish uniqueness of the MFG solutions under monotonicity conditions of Lasry–Lions type. Finally, we show in a setting with finite-dimensional interaction that such solutions induce approximate Nash equilibria for the N-player game with vanishing error as $N \to \infty$.

Résumé. La classe des jeux à champs moyen avec absorption est une classe de jeux qui a été introduite dans (*Ann. Appl. Probab.* **28** (2018) 2188–2242). Elle peut être vue comme la limite naturelle de jeux différentiels stochastiques symétriques avec un grand nombre de joueurs qui interagissent entre eux par un champ moyen et qui quittent le jeu dès que leur état touche une frontière donnée.

Dans cet article, on poursuit l'étude de ces jeux, en les étendant dans deux directions. Premièrement, on considère des variables d'état et des coûts avec une dépendance très générale des sous-probabilités empiriques (non normalisées) des états des survivants. Ceci inclut le cas particulièrement important où l'interaction entre les joueurs se fait par la mesure empirique des survivants ainsi que par la proportion des joueurs absorbés au cours du temps. Deuxièmement, l'hypothèse des coefficients et coûts bornés a été considérablement relaxée en la remplaçant par celle de croissance linéaire, ce qui permet d'avoir des dynamiques plus réalistes pour les variables d'état. On montre l'existence des solutions du jeu à champ moyen aussi bien pour des contrôles strictes que relaxés. On établit l'unicité de la solution sous des conditions de monotonie à la Lasry–Lions. Enfin, dans un cadre d'interaction fini-dimensionnelle, on montre que ces solutions induisent des équilibres de Nash approchés pour le jeu à N joueurs avec une erreur qui tend vers zéro quand $N \to \infty$.

MSC2020 subject classifications: 60B10; 60K35; 91A06; 93E20

Keywords: Nash equilibrium; Mean-field game; Absorbing boundary; McKean–Vlasov limit; Controlled martingale problem; Relaxed control

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Barak–Erdős graphs and the infinite-bin model

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Abstract. A Barak–Erdős graph is a directed acyclic version of the Erdős–Rényi random graph. It is obtained by performing independent bond percolation with parameter p on the complete graph with vertices $\{1,\dots,n\}$, in which the edge between two vertices $i<j$ is directed from i to j. The length of the longest path in this graph grows linearly with the number of vertices, at rate $C(p)$. In this article, we use a coupling between Barak–Erdős graphs and infinite-bin models to provide explicit estimates on $C(p)$. More precisely, we prove that the front of an infinite-bin model grows at linear speed, and that this speed can be obtained as the sum of a series. Using these results, we prove the analyticity of C for $p>1/2$, and compute its power series expansion. We also obtain the first two terms of the asymptotic expansion of C as $p\to 0$, using a coupling with branching random walks with selection.

Résumé. Un graphe de Barak–Erdős est une version dirigée et sans cycle du graphe aléatoire d'Erdős–Rényi. Ce graphe est construit en réalisant une percolation par arêtes de paramètre p sur le graphe complet d'ensemble de sommets $\{1,\dots,n\}$, tel que l'arête entre deux sommets $i<j$ est orientée de i vers j. La longueur du plus long chemin dans ce graphe croît linéairement avec le nombre n de sommets, à vitesse $C(p)$. Dans cet article, nous utilisons un couplage entre le graphe de Barak–Erdős et un modèle infini d'urnes pour obtenir des estimations explicites pour $C(p)$. Plus précisément, on montre que le front d'un modèle infini d'urne croît à vitesse linéaire, et que cette vitesse peut être obtenue comme la somme d'une série. Grâce à ces résultats, on montre que la fonction C est analytique pour $p>1/2$, et on obtient son développement en série entière autour de $p=1$. Nous calculons également les deux premiers termes du développement limité de C au voisinage de $p=0$, grâce à un couplage avec des marches aléatoires branchantes avec sélection.

MSC2020 subject classifications: Primary 05C20; 05C38; 70F45; secondary 60F15; 60J10; 60J85; 60K30

Keywords: Barak–Erdős graph; Infinite-bin model; Branching random walk; Selection; Coupling; Stochastic ordered graph

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Penalising transmission to hubs in scale-free spatial random graphs

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Abstract. We study the spread of information in finite and infinite inhomogeneous spatial random graphs. We assume that each edge has a transmission cost that is a product of an i.i.d. random variable L and a penalty factor: edges between vertices of expected degrees w_1 and w_2 are penalised by a factor of $(w_1w_2)^\mu$ for all $\mu > 0$. We study this process for scale-free percolation, for (finite and infinite) Geometric Inhomogeneous Random Graphs, and for Hyperbolic Random Graphs, all with power law degree distributions with exponent $\tau > 1$. For $\tau < 3$, we find a threshold behaviour, depending on how fast the cumulative distribution function of L decays at zero. If it decays at most polynomially with exponent smaller than $(3-\tau)/(2\mu)$ then explosion happens, i.e., with positive probability we can reach infinitely many vertices with finite cost (for the infinite models), or reach a linear fraction of all vertices with bounded costs (for the finite models). On the other hand, if the cdf of L decays at zero at least polynomially with exponent larger than $(3-\tau)/(2\mu)$, then no explosion happens. This behaviour is arguably a better representation of information spreading processes in social networks than the case without penalising factor, in which explosion always happens unless the cdf of L is doubly exponentially flat around zero. Finally, we extend the results to other penalty functions, including arbitrary polynomials in w_1 and w_2. In some cases the interesting phenomenon occurs that the model changes behaviour (from explosive to conservative and vice versa) when we reverse the role of w_1 and w_2. Intuitively, this could corresponds to reversing the flow of information: gathering information might take much longer than sending it out.

Résumé. Nous étudions la propagation de l'information dans des graphes aléatoires spatiaux finis et infinis. Nous supposons que chaque arête a un coût de transmission qui est un produit d'une variable aléatoire L i.i.d. et d'un facteur de pénalité : les arêtes entre sommets de degrés moyens w_1 et w_2 sont pénalisées par un facteur de $(w_1w_2)^\mu$ pour tout $\mu > 0$. Nous étudions ce processus pour la percolation sans échelle, pour des graphes aléatoires géométriques inhomogènes (finis ou non), et pour des graphes aléatoires hyperboliques, tous avec des degrés suivant des lois de puissance d'exposant $\tau > 1$. Pour $\tau < 3$, nous déterminons un seuil, dépendant de la vitesse à laquelle la fonction de répartition de L tend vers 0 en 0. Si cette vitesse est au plus polynomiale avec exposant plus petit que $(3-\tau)/(2\mu)$ alors on a l'explosion, au sens où avec probabilité strictement positive, une infinité de sommets peuvent être atteints à coût fini (pour le modèle infini), ou un fraction linéaire de l'ensemble des sommets peut être atteint à coût borné (pour le modèle fini). Par ailleurs, si la vitesse de décroissance en 0 de la fonction de répartition de L est au moins polynomiale avec un exposant plus grand que $(3-\tau)/(2\mu)$, alors on n'a pas d'explosion. Ce comportement des processus de transmission d'information dans les réseaux sociaux est plus réaliste que dans les modèles sans facteur de pénalisation, où l'explosion a toujours lieu à moins que la fonction de répartition de L n'ait une décroissance en 0 doublement exponentielle.

Enfin, nous étendons ces résultats à d'autres fonctions de pénalité, incluant des polynômes arbitraires en w_1, w_2. Dans certains cas, nous observons l'intéressant phénomène que le modèle change de comportement (d'explosif à conservatif et vice-versa) lorsque l'on échange les rôles de w_1 et w_2. Intuitivement, cela pourrait correspondre à un retournement du flot d'information : acquérir de l'information peut prendre plus de temps que de l'envoyer.

MSC2020 subject classifications: 05C80; 05C82; 60C05

Keywords: SI epidemic; Information spread; Explosive first passage percolation; Scale-free percolation; Geometric inhomogeneous random graphs; Hyperbolic random graphs

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Fluctuations for the partition function of Ising models on Erdös–Rényi random graphs

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Abstract. We analyze Ising/Curie–Weiss models on the Erdős–Rényi graph with N vertices and edge probability $p = p(N)$ that were introduced by Bovier and Gayrard (*J. Stat. Phys.* **72** (3–4) (1993) 643–664) and investigated in (*J. Stat. Phys.* **177** (1) (2019) 78–94) and (Kabluchko, Löwe and Schubert (2019)). We prove Central Limit Theorems for the partition function of the model and – at other decay regimes of $p(N)$ – for the logarithmic partition function. We find critical regimes for $p(N)$ at which the behavior of the fluctuations of the partition function changes.

Résumé. Nous analysons les modéles d'Ising/Curie–Weiss sur le graphe Erdős–Rényi avec N sommets et probabilité d'arête $p = p(N)$ introduits par Bovier et Gayrard (*J. Stat. Phys.* **72** (3–4) (1993) 643–664) et étudié dans (*J. Stat. Phys.* **177** (1) (2019) 78–94) et (Kabluchko, Löwe and Schubert (2019)). Nous montrons des théorèmes limite central pour la fonction de partition du modèle et – à autres régimes de $p(N)$ – pour la fonction de partition logarithmique. Nous trouvons des régimes critiques pour $p(N)$ en lequels le comportement des fluctuations de la fonction de partition change.

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Keywords: Ising model; Dilute Curie–Weiss model; Fluctuations; Partition function; Central Limit Theorem; Random graphs

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Mean-field Langevin dynamics and energy landscape of neural networks

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Abstract. Our work is motivated by a desire to study the theoretical underpinning for the convergence of stochastic gradient type algorithms widely used for non-convex learning tasks such as training of neural networks. The key insight, already observed in (Mei, Montanari and Nguyen (2018); Chizat and Bach (2018); Rotskoff and Vanden-Eijnden (2018)), is that a certain class of the finite-dimensional non-convex problems becomes convex when lifted to infinite-dimensional space of measures. We leverage this observation and show that the corresponding energy functional defined on the space of probability measures has a unique minimiser which can be characterised by a first-order condition using the notion of linear functional derivative. Next, we study the corresponding gradient flow structure in 2-Wasserstein metric, which we call Mean-Field Langevin Dynamics (MFLD), and show that the flow of marginal laws induced by the gradient flow converges to a stationary distribution, which is exactly the minimiser of the energy functional. We observe that this convergence is exponential under conditions that are satisfied for highly regularised learning tasks. Our proof of convergence to stationary probability measure is novel and it relies on a generalisation of LaSalle's invariance principle combined with HWI inequality. Importantly, we assume neither that interaction potential of MFLD is of convolution type nor that it has any particular symmetric structure. Furthermore, we allow for the general convex objective function, unlike, most papers in the literature that focus on quadratic loss. Finally, we show that the error between finite-dimensional optimisation problem and its infinite-dimensional limit is of order one over the number of parameters.

Résumé. L'objectif de nos travaux est d'étudier le fondement théorique pour la convergence des algorithmes du type gradient stochastique, qui sont très souvent utilisés dans les problèmes d'apprentissage non-convexe, e.g. calibrer un réseau de neurones. L'observation clé, qui a déjà été remarquée dans (Mei, Montanari and Nguyen (2018); Chizat and Bach (2018); Rotskoff and Vanden-Eijnden (2018)), est qu'une certaine classe de problèmes non-convexes fini-dimensionnels devient convexe une fois injectée dans l'espace des mesures de probabilité. À l'aide de cette observation nous montrons que la fonction d'énergie correspondante définie dans l'espace des mesures de probabilité a un unique minimiser qui peut être caractérisé par une condition de premier ordre en utilisant la notion de dérivée fonctionnelle. Par la suite, nous étudions la structure de flux de gradient avec la métrique de 2-Wasserstein, que nous appelons la dynamique de Langevin au champs moyen (MFLD), et nous montrons que la loi marginale du flux de gradient converge vers une loi stationnaire qui correspond au minimiser de la même fonction d'énergie précédente. Sous certaines conditions de régularité du probléme initial, la convergence a lieu à une vitesse exponentielle. Nos preuves de la convergence vers la loi stationnaire est nouvelle, qui reposent sur le principe d'invariance de LaSalle et l'inégalité HWI. Remarquons que nous ne supposons pas que l'interaction potentielle de MFLD soit du type convolution ou symétrique. De plus, nos résultats s'appliquent aux fonctions d'objectif convexes générales contrairement aux beaucoup d'articles dans la littérature qui se limitent aux fonctions quadratiques. Enfin, nous montrons que la différence entre le probléme initial d'optimisation fini-dimensionnel et sa limite dans l'espace des mesures de probabilité est de l'ordre d'un sur le nombre de paramètres.

MSC2020 subject classifications: 60H30; 37M25

Keywords: Mean-field Langevin dynamics; Gradient flow; Neural networks

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Stable Lévy processes in a cone

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Abstract. Bañuelos and Bogdan (*Potential Anal.* **21** (3) (2004) 263–288) and Bogdan et al. (*Electron. J. Probab.* **23** (2018) 11) analyse the asymptotic tail distribution of the first time a stable (Lévy) process in dimension $d \geq 2$ exits a cone. We use these results to develop the notion of a stable process conditioned to remain in a cone as well as the the notion of a stable process conditioned to absorb continuously at the apex of a cone (without leaving the cone). As self-similar Markov processes, we examine some of their fundamental properties through the lens of its Lamperti–Kiu decomposition. In particular we are interested to understand the underlying structure of the Markov additive process (MAP) that drives such processes. Through the interrogation of the underlying MAP, we are able to provide an answer by example to the open question: If the modulator of a MAP has a stationary distribution, under what conditions does its ascending ladder MAP have a stationary distribution?

With the help of an analogue of the Riesz–Bogdan–Żak transform (cf. Bogdan and Żak (*J. Theoret. Probab.* **19** (1) (2006) 89–120), Kyprianou (*Electron. J. Probab.* **21** (2016) 23), Alili et al. (*Electron. J. Probab.* **22** (2017) 20)) as well as Hunt–Nagasawa duality theory, we show how the two forms of conditioning are dual to one another. Moreover, in the sense of Rivero (*Bernoulli* **11** (3) (2005) 471–509; *Bernoulli* **13** (4) (2007) 1053–1070) and Fitzsimmons (*Electron. Commun. Probab.* **11** (2006) 230–241), we construct the null-recurrent extension of the stable process killed on exiting a cone, showing that it again remains in the class of self-similar Markov processes. Aside from the Riesz–Bogdan–Żak transform and Hunt–Nagasawa duality, an unusual combination of the Markov additive renewal theory of e.g. Alsmeyer (*Stochastic Process. Appl.* **50** (1) (1994) 37–56) as well as the boundary Harnack principle (see e.g. *Electron. J. Probab.* **23** (2018) 11) play a central role to the analysis.

In the spirit of several very recent works (see *Stochastic Process. Appl.* **129** (3) (2019) 954–977; *Electron. J. Probab.* **21** (2016) 23; *Ann. Inst. Henri Poincaré Probab. Stat.* **54** (1) (2018) 343–362; *Potential Anal.* **53** (2020) 1347–1375; *ALEA Lat. Am. J. Probab. Math. Stat.* **15** (1) (2018) 617–690; *Ann. Probab.* **48** (3) (2020) 1220–1265), the results presented here show that previously unknown results of stable processes, which have long since been understood for Brownian motion, or are easily proved for Brownian motion, become accessible by appealing to the properties of the stable process as a self-similar Markov process, combined with its special status as a Lévy processes having semi-tractable potential analysis.

Résumé. Bañuelos et Bogdan (*Potential Anal.* **21** (3) (2004) 263–288) et Bogdan et al. (*Electron. J. Probab.* **23** (2018) 11) ont etudié la queue de distribution du temps pour qu'un processus stable (Lévy) de dimension $d \geq 2$ sorte d'un cône. Nous utilisons ces résultats pour développer le concept de processus stable conditionné à rester dans un cône ainsi que celui du processus stable conditionné à être absorbé continûement au sommet d'un cône (sans sortir de celui-ci). En tant que processus de Markov auto-similaires, nous examinons certaines de leurs propriétés fondamentales en utilisant la décomposition de Lamperti–Kiu. En particulier, nous sommes intéressés à comprendre la structure sous-jacente du processus de Markov additif (Markov Additive Process – MAP) qui dirige des tels processus. En étudiant le MAP sous-jacent, nous sommes en mesure de répondre aux questions ouvertes, telles que : si le modulateur d'un MAP a une distribution stationnaire, dans quelles conditions le processus d'échelle ascendant associé, qui est aussi un MAP, a-t-il une distribution stationnaire ?

À l'aide d'un analogue de la transformation Riesz–Bogdan–Zak (cf. Bogdan and Żak (*J. Theoret. Probab.* **19** (1) (2006) 89–120), Kyprianou (*Electron. J. Probab.* **21** (2016) 23), Alili et al. (*Electron. J. Probab.* **22** (2017) 20)) ainsi que la théorie de la dualité de Hunt–Nagasawa, nous montrons comment les deux formes de conditionnement sont duales l'une de l'autre. De plus, au sens de Rivero (*Bernoulli* **11** (3) (2005) 471–509 ; *Bernoulli* **13** (4) (2007) 1053–1070) et Fitzsimmons (*Electron. Commun. Probab.* **11** (2006) 230–241), nous construisons l'extension nulle-récurrente du processus stable tué à la sortie d'un cône, montrant que celui-ci appartient encore à la classe des processus de Markov auto-similaire. Outre la transformation de Riesz–Bogdan–Zak et la dualité de Hunt–Nagasawa, une combinaison inhabituelle de la théorie du renouvellement Markovien additif, par ex. Alsmeyer (*Stochastic Process. Appl.* **50** (1) (1994) 37–56), ainsi que le principe de frontière de Harnack (voir par exemple *Electron. J. Probab.* **23** (2018) 11) jouent un rôle central dans notre analyse.

Dans l'esprit de plusieurs travaux très récents (voir *Stochastic Process. Appl.* **129** (3) (2019) 954–977 ; *Electron. J. Probab.* **21** (2016) 23 ; *Ann. Inst. Henri Poincaré Probab. Stat.* **54** (1) (2018) 343–362 ; *Potential Anal.* **53** (2020) 1347–1375 ; *ALEA Lat. Am.*

J. Probab. Math. Stat. **15** (1) (2018) 617–690 ; *Ann. Probab.* **48** (3) (2020) 1220–1265), les résultats présentés ici montrent que des résultats jusqu'à maintenant inconnus pour les processus stables, deviennent accessibles en faisant appel aux propriétés du processus stable, en tant que processus de Markov auto-similaire, en conjugaison avec le fait qu'il s'agit des processus de Lévy avec une théorie du potentiel accessible. Ces résultats sont peut être bien compris depuis longtemps pour le mouvement Brownien ou bien peuvent être facilement prouvés pour celui-ci, notre approche permets donc de combler cette lacune dans la théorie des processus stables.

MSC2020 subject classifications: Primary 60H20; 60J99; 60J80; secondary 60G52

Keywords: Stable processes; Entrance law; Kelvin transform; Duality; Lévy processes

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Extreme eigenvalue statistics of m-dependent heavy-tailed matrices

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Abstract. We analyze the largest eigenvalue statistics of m-dependent heavy-tailed Wigner matrices as well as the associated sample covariance matrices having entry-wise regularly varying tail distributions with parameter $\alpha \in (0, 4)$. Our analysis extends results in the previous literature for the corresponding random matrices with independent entries above the diagonal, by allowing for m-dependence between the entries of a given matrix. We prove that the limiting point process of extreme eigenvalues is a Poisson cluster process.

Résumé. Nous analysons les plus grandes valeurs propres d'une matrice de Wigner avec entrées m-dépendantes et à queue lourde, de même que pour une matrice de covariance associée avec entrées de variation régulière de paramètre $\alpha \in (0, 4)$. Notre analyse étend les résultats existants pour ces matrices aléatoires avec entrées indépendantes à des entrées m-dépendantes. Nous prouvons que le processus ponctuel limite des plus grandes valeurs propres est un processus de Poisson groupé.

MSC2020 subject classifications: Primary 60B20; secondary 60F05; 60F10; 60G10; 60G55; 60G70

Keywords: Dependent random matrices; Largest eigenvalue; Heavy-tailed random matrices; Poisson cluster process; Marked Poisson process; Regular variation; Wigner matrix; Sample covariance matrix

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Coexistence of competing first passage percolation on hyperbolic graphs

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Abstract. We study a natural growth process with competition, which was recently introduced to analyze MDLA, a challenging model for the growth of an aggregate by diffusing particles. The growth process consists of two first-passage percolation processes FPP_1 and FPP_λ, spreading with rates 1 and $\lambda > 0$ respectively, on a graph G. FPP_1 starts from a single vertex at the origin o, while the initial configuration of FPP_λ consists of infinitely many *seeds* distributed according to a product of Bernoulli measures of parameter $\mu > 0$ on $V(G) \setminus \{o\}$. FPP_1 starts spreading from time 0, while each seed of FPP_λ only starts spreading after it has been reached by either FPP_1 or FPP_λ. A fundamental question in this model, and in growth processes with competition in general, is whether the two processes coexist (i.e., both produce infinite clusters) with positive probability. We show that this is the case when G is vertex transitive, non-amenable and hyperbolic, in particular, for any $\lambda > 0$ there is a $\mu_0 = \mu_0(G, \lambda) > 0$ such that for all $\mu \in (0, \mu_0)$ the two processes coexist with positive probability. This is the first non-trivial instance where coexistence is established for this model. We also show that FPP_λ produces an infinite cluster almost surely for any positive λ, μ, establishing fundamental differences with the behavior of such processes on $\mathbb{Z}^d$.

Résumé. Nous étudions un processus de croissance naturel avec compétition, qui a été récemment introduit pour analyser le MDLA, un modèle bien connu et difficile de croissance d'un agrégat par des particules diffusives. Le modèle de croissance considéré est constitué de deux processus de percolation de premier passage FPP_1 et FPP_λ, se propageant à taux 1 et $\lambda > 0$ respectivement, sur un graphe G. Le processus FPP_1 débute par un simple sommet à l'origine, alors que la condition initiale de FPP_λ consiste en une infinité de *germes* distribués selon un produit de lois de Bernoulli de paramètre $\mu > 0$ sur $V(G) \setminus \{o\}$. Le processus FPP_1 débute au temps 0 alors que les germes de FPP_λ ne se propagent qu'après avoir été atteints par FPP_1 ou FPP_λ. Une question fondamentale dans ce modèle, et dans les modèles de croissance en compétition en général, est savoir si les deux processus peuvent coexister (c'est-à-dire que les deux produisent des composantes infinies) avec probabilité strictement positive. Nous montrons que c'est le cas lorsque G est transitif sur les arêtes, non-moyennable et hyperbolique, en particulier, pour tout $\lambda > 0$ il existe $\mu_0 = \mu_0(G, \lambda) > 0$ tel que pour tout $\mu \in (0, \mu_0)$ les deux processus coexistent avec probabilité strictement positive. C'est la première fois qu'un résultat de coexistence non trivial est établi pour ce modèle. Nous montrons aussi que FPP_λ produit une composante infinie presque surement pour tout λ, μ positifs, montrant des différences fondamentales avec le comportement de tels processus sur $\mathbb{Z}^d$.

MSC2020 subject classifications: 60K35; 82B43; 82C22

Keywords: First passage percolation; First passage percolation in hostile environment; Hyperbolic graphs; Non-amenable graphs; Competition; Coexistence; Two-type Richardson model

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Last passage percolation in an exponential environment with discontinuous rates

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Abstract. We prove a strong law of large numbers for directed last passage times in an independent but inhomogeneous exponential environment. Rates for the exponential random variables are obtained from a discretisation of a speed function that may be discontinuous on a locally finite set of discontinuity curves. The limiting shape is cast as a variational formula that maximises a certain functional over a set of weakly increasing curves.

Résumé. On montre une loi des grands nombres pour les temps de dernier passage dirigé dans un environnement indépendant mais inhomogène et exponentiel. Les taux des variables exponentielles sont obtenues a partir d'une discretisation d'une fonction de vitesse macroscopique qui pourrait être discontinue sur un ensemble localement fini des courbes de discontinuité. La forme à la limite est déterminée par une formule des variations qui maximise une certaine fonctionnel sur un ensemble des courbes faiblement croissantes.

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Keywords: Last passage time; Discontinuous percolation; Discontinuous environment; Two-phase models; Multi-phase models; Variational formula; Corner growth model; Shape theorem

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Russo–Seymour–Welsh estimates for the Kostlan ensemble of random polynomials

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Abstract. Beginning with the predictions of Bogomolny–Schmit for the random plane wave, in recent years the deep connections between the level sets of smooth Gaussian random fields and percolation have become apparent. In classical percolation theory a key input into the analysis of global connectivity are scale-independent bounds on crossing probabilities in the critical regime, known as Russo–Seymour–Welsh (RSW) estimates. Similarly, establishing RSW-type estimates for the nodal sets of Gaussian random fields is a major step towards a rigorous understanding of these relations.

The Kostlan ensemble is an important model of Gaussian homogeneous random polynomials. The nodal set of this ensemble is a natural model for a 'typical' real projective hypersurface, whose understanding can be considered as a statistical version of Hilbert's 16th problem. In this paper we establish RSW-type estimates for the nodal sets of the Kostlan ensemble in dimension two, providing a rigorous relation between random algebraic curves and percolation. The estimates are uniform with respect to the degree of the polynomials, and are valid on all relevant scales; this, in particular, resolves an open question raised recently by Beffara–Gayet. More generally, our arguments yield RSW estimates for a wide class of Gaussian ensembles of smooth random functions on the sphere or the flat torus.

Résumé. Partant des prédictions de Bogomolny–Schmit pour les ondes planaires aléatoires, dans les années récentes des relations profondes sont apparues entre les lignes de niveau des champs aléatoires Gaussiens réguliers et la percolation. En théorie de la percolation, un ingrédient clé dans l'analyse de la connectivité globale est la famille des bornes indépendantes en échelle sur les probabilités de croisement dans le régime critique, connues sous le nom d'estimées de Russo–Seymour–Welsh (RSW). De la même façon, établir des estimées du type RSW pour les ensembles nodaux des champs aléatoires Gaussiens est une étape majeure dans la compréhension rigoureuse de ces relations.

L'ensemble de Kostlan est un modèle important de polynômes aléatoires homogènes Gaussiens. L'ensemble nodal des polynômes de Kostlan est un modèle naturel pour une hypersurface projective réelle typique, dont la compréhension peut être vu comme une version statistique du 16ème problème de Hilbert. Dans cet article, nous établissons une estimées du type RSW pour les ensembles nodaux des polynômes de Kostlan en dimension 2, montrant ainsi une relation rigoureuse entre les courbes algébriques aléatoires et la percolation. Les estimées sont uniformes en le degré du polynôme, et sont valables dans toutes les échelles pertinentes ; ceci, en particulier, résout la question posée récemment par Beffara–Gayet. Plus généralement, nos arguments conduisent à des estimées RSW pour une large classe d'ensembles Gaussiens de fonctions régulières aléatoires sur la sphère ou sur le tore plat.

MSC2020 subject classifications: 60G15; 60K35; 30C15

Keywords: Kostlan ensemble; Gaussian field; Nodal set; Percolation; Russo–Seymour–Welsh estimates

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Once reinforced random walk on $\mathbb{Z} \times \Gamma$

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Abstract. We revisit Vervoort's unpublished paper (Vervoort (2002)) on the once reinforced random walk, and prove that this process is recurrent on any graph of the form $\mathbb{Z} \times \Gamma$, with Γ a finite graph, for sufficiently large reinforcement parameter. We also obtain a shape theorem for the set of visited sites, and show that the fluctuations around this shape are of polynomial order. The proof involves sharp general estimates on the time spent on subgraphs of the ambiant graph which might be of independent interest.

Résumé. Nous revisitons un article de Vervoort (Vervoort (2002)), jamais publié, sur la marche une-fois-renforcée, ou *once reinforced random walk*, et nous prouvons que ce processus est récurrent sur tout graphe de la forme $\mathbb{Z} \times \Gamma$, où Γ est un graphe fini, et pour un paramètre de renforcement suffisamment grand. Nous obtenons également un théorème de forme pour l'ensemble des sites visités, et prouvons que les fluctuations sont d'ordre polynomial. La preuve utilise des estimées précises et générales sur le temps passé dans des sous-graphes du graphe ambiant, ce qui pourrait être intéressant par soi-même.

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Keywords: Recurrence; Reinforced random walk; Self-interacting random walk; Shape theorem

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Quenched invariance principle for long range random walks in balanced random environments

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Abstract. We establish via a probabilistic approach the quenched invariance principle for a class of long range random walks in independent (but not necessarily identically distributed) balanced random environments, with the transition probability from x to y on average being comparable to $|x-y|^{-(d+\alpha)}$ with $\alpha \in (0,2]$. We use the martingale property to estimate exit time from balls and establish tightness of the scaled processes, and apply the uniqueness of the martingale problem to identify the limiting process. When $\alpha \in (0,1)$, our approach works even for non-balanced cases. When $\alpha = 2$, under a diffusive with the logarithmic perturbation scaling, we show that the limit of scaled processes is a Brownian motion.

Résumé. Nous établissons par une approche probabiliste le principe d'invariance à environnement gelé pour une classe de marches aléatoires à longue distance dans des environnements aléatoires équilibrés indépendants (mais pas nécessairement distribués de manière identique), la probabilité de transition de x à y en moyenne étant comparable à $|x-y|^{-(d+\alpha)}$ avec $\alpha \in (0,2]$. Nous utilisons la propriété martingale pour estimer le temps de sortie des balles et établir l'étanchéité des processus renormalisés, et appliquer l'unicité du problème de la martingale pour identifier le processus limite. Lorsque $\alpha \in (0,1)$, notre approche fonctionne même pour les cas non équilibrés. Lorsque $\alpha = 2$, sous un diffusif à l'échelle de perturbation logarithmique, nous montrons que la limite des processus renormalisés est un mouvement brownien.

MSC2020 subject classifications: 60G51; 60G52; 60J25; 60J75

Keywords: Long range random walk; Balanced random environment; Martingale problem

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A family of fractional diffusion equations derived from stochastic harmonic chains with long-range interactions

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Abstract. We consider one-dimensional infinite chains of harmonic oscillators with stochastic perturbations and long-range interactions which have polynomial decay rate $|x|^{-\theta}$, $x \to \infty$, $\theta > 1$, where $x \in \mathbb{Z}$ is the interaction range. We prove that if $2 < \theta \leq 3$, then the time evolution of the macroscopic thermal energy distribution is superdiffusive and governed by a fractional diffusion equation with exponent $\frac{3}{7-\theta}$, while if $\theta > 3$, then the exponent is $\frac{3}{4}$. The threshold is $\theta = 3$ because the derivative of the dispersion relation diverges as $k \to 0$ when $\theta \leq 3$.

Résumé. Nous considérons une chaîne infinie unidimensionnelle d'oscillateurs harmoniques avec des perturbations stochastiques et une interaction à longue portée avec une décroissance polynomiale d'ordre $|x|^{-\theta}$, $x \to \infty$, $\theta > 1$, où x est la longueur d'interaction. Nous montrons que si $2 < \theta \leq 3$, l'évolution en temps de la distribution d'énergie thermique macroscopique est sur diffusive et régie par une équation de diffusion fractionnaire avec exposant $\frac{3}{7-\theta}$, alors que si $\theta > 3$, l'exposant est $\frac{3}{4}$. Le seuil est $\theta = 3$ car la dérivée de la relation de dispersion diverge lorsque $k \to 0$ quand $\theta \leq 3$.

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Keywords: Harmonic chain; Long-range interaction; Wigner distribution; Fractional diffusion equation

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Infinite-dimensional regularization of McKean–Vlasov equation with a Wasserstein diffusion

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Abstract. Much effort has been spent in recent years on restoring uniqueness of McKean–Vlasov SDEs with non-smooth coefficients. As a typical instance, the velocity field b is assumed to be bounded and measurable in its space variable and Lipschitz-continuous with respect to the distance in total variation in its measure variable, as shown e.g. in the works of Jourdain and Mishura-Veretennikov. In contrast with those works, we consider in this paper a Fokker–Planck equation driven by an infinite-dimensional noise, inspired by the diffusion models on the Wasserstein space studied by Konarovskyi and von Renesse. We prove via Girsanov's Theorem that there exists a unique weak solution to that equation for a drift function that might be only bounded and measurable in its measure argument, provided that a trade-off is respected between the regularity in the finite-dimensional component and the regularity in the measure argument. In this regard, we show that the higher the regularity of b with respect to its space variable is, the lower regularity we have to assume on b with respect to its measure variable in order to restore uniqueness in a weak sense.

Résumé. Le problème de la restauration de l'unicité des EDS de McKean–Vlasov avec coefficients non réguliers a fait l'objet de beaucoup de contributions ces dernières années. Le champ de vitesse b y est typiquement supposé borné et mesurable en la variable d'espace et lipschitzien par rapport à la distance en variation totale en la variable de mesure, comme par exemple dans les travaux de Jourdain et de Mishura-Veretennikov. Contrairement à ces travaux, nous considérons dans cet article une équation de Fokker–Planck dirigée par un bruit infini-dimensionnel, inspiré par les modèles de processus de diffusion sur l'espace de Wasserstein étudiés par Konarovskyi and von Renesse. Nous prouvons à l'aide du théorème de Girsanov que cette équation admet une unique solution faible lorsque le coefficient de dérive b est uniquement borné et mesurable en la variable de mesure, à condition qu'en contrepartie sa régularité en la variable finie-dimensionelle soit plus élevée. En ce sens, nous montrons ensuite qu'afin d'obtenir la restauration de l'unicité au sens faible, la régularité que nous avons à imposer à b en la variable de mesure est d'autant plus faible que b est plus régulière en espace.

MSC2020 subject classifications: Primary 60H10; 60H15; secondary 60K35; 60J60; 35Q83

Keywords: Wasserstein diffusion; McKean–Vlasov equation; Fokker–Planck equation; Regularization properties; Restoration of uniqueness; Interacting particle system; Coalescing particles; Brownian sheet

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Porous media equations with multiplicative space–time white noise

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Abstract. The existence of martingale solutions for stochastic porous media equations driven by nonlinear multiplicative space–time white noise is established in spatial dimension one. The Stroock–Varopoulos inequality is identified as a key tool in the derivation of the corresponding estimates.

Résumé. L'existence d'une solution martingale pour l'équation stochastique des milieux poreux, dirigée par un bruit non-linéaire multiplicatif en espace et en temps, est établie dans le cas spatial unidimensionnel. L'inégalité de Stroock–Varopoulos est identifiée comme un outil clé dans l'obtention des estimées correspondantes.

MSC2020 subject classifications: Primary 60H15; secondary 35K65; 35K59

Keywords: Stochastic PDEs; Porous media equations

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The elliptic stochastic quantization of some two dimensional Euclidean QFTs

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Abstract. We study a class of elliptic SPDEs with additive Gaussian noise on $\mathbb{R}^2 \times M$, with M a d-dimensional manifold equipped with a positive Radon measure, and a real-valued non linearity given by the derivative of a smooth potential V, convex at infinity and growing at most exponentially. For quite general coefficients and a suitable regularity of the noise we obtain, via the dimensional reduction principle discussed in our previous paper (*Ann. Probab.* **48** (2020) 1693–1741), the identity between the law of the solution to the SPDE evaluated at the origin with a Gibbs type measure on an abstract Wiener space over M. The results are then applied to the elliptic stochastic quantization equation for the scalar field with polynomial interaction over $\mathbb{T}^2$, and with exponential interaction over $\mathbb{R}^2$ (known also as Høegh-Krohn or Liouville model in the literature). In particular for the exponential interaction case, the existence and uniqueness properties of solutions to the elliptic equation over $\mathbb{R}^{2+2}$ is derived as well as the dimensional reduction for the values of the "charge parameter" $\sigma = \frac{\alpha}{2\sqrt{\pi}} < \sqrt{4(8-4\sqrt{3})\pi} \simeq \sqrt{4.29\pi}$, for which the model has an Euclidean invariant, reflection positive probability measure (hence also permitting to get the corresponding relativistic invariant model on the two dimensional Minkowski space).

Résumé. Nous étudions une classe d'EDPS elliptiques avec bruit gaussien additif sur $\mathbb{R}^2 \times M$, où M est une variété de dimension d équipée d'une mesure de Radon positive, et avec une non-linéarité à valeurs réelles donnée par la dérivée d'un potentiel lisse V, convexe à l'infini et de croissance au plus exponentielle. Sous des conditions assez générales sur les coefficients et sur la régularité du bruit, nous obtenons par le principe de réduction dimensionnelle discuté dans notre précédent article (*Ann. Probab.* **48** (2020) 1693–1741), l'identité entre la loi de la solution de l'EDPS évaluée à l'origine et une mesure de type Gibbs sur un espace de Wiener abstrait sur M. Les résultats sont ensuite appliqués à l'équation de quantisation elliptique stochastique pour un champ scalaire avec des interactions polynomiales sur $\mathbb{T}^2$, et avec des interactions exponentielles sur $\mathbb{R}^2$ (un modèle connu dans la littérature sous le nom de modèle de Høegh-Krohn ou de modèle de Liouville). En particulier, pour le cas de l'interaction exponentielle, on obtient les propriétés d'existence et d'unicité des solutions à l'équation elliptique sur $\mathbb{R}^{2+2}$, ainsi que la réduction dimensionnelle pour les valeurs du « paramètre de charge » $\sigma = \frac{\alpha}{2\sqrt{\pi}} < \sqrt{4(8-4\sqrt{3})\pi} \simeq \sqrt{4.29\pi}$, pour lesquels le modèle a une mesure de probabilité euclidienne invariante et réflexion-positive (ce qui permet ainsi d'obtenir le modèle invariant relativiste correspondant sur l'espace de Minkowski de dimension 2).

MSC2020 subject classifications: Primary 60H15; secondary 81T08; 81T40

Keywords: Stochastic quantization; Elliptic stochastic partial differential equations; Dimensional reduction; Euclidean quantum field theory; Exponential interaction

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