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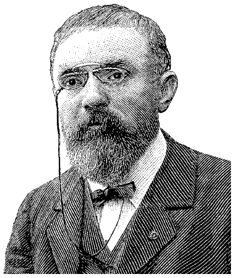

ANNALES DE L'INSTITUT HENRI POINCARÉ
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Scaling limits for the critical Fortuin–Kasteleyn model on a random planar map I: Cone times

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Abstract. Sheffield (2011) introduced an inventory accumulation model which encodes a random planar map decorated by a collection of loops sampled from the critical Fortuin–Kasteleyn (FK) model. He showed that a certain two-dimensional random walk associated with the infinite-volume version of the model converges in the scaling limit to a correlated planar Brownian motion. We improve on this scaling limit result by showing that the times corresponding to FK loops (or "flexible orders") in the inventory accumulation model converge in the scaling limit to the $\pi/2$-cone times of the correlated Brownian motion. This statement implies a scaling limit result for the joint law of the areas and boundary lengths of the bounded complementary connected components of the FK loops on the infinite-volume planar map. In light of the encoding of Duplantier, Miller, and Sheffield (2014), the limiting object coincides with the joint law of the areas and boundary lengths of the bounded complementary connected components of a collection of CLE loops on an independent Liouville quantum gravity surface.

Résumé. Sheffield a introduit en 2011 un modèle d'accumulation de stocks, qui code une carte planaire aléatoire décorée par une collection de boucles, échantillonnée selon le modèle de percolation de Fortuin–Kasteleyn (FK) critique. Il a démontré que certaines marches aléatoires planes associées au modèle en volume infini convergent dans la limite d'échelle vers un mouvement brownien plan corrélé. Nous améliorons ce résultat de limite d'échelle en montrant que les temps correspondant aux boucles FK (ou « commandes flexibles ») dans le modèle d'accumulation de stocks convergent dans la limite d'échelle vers les temps de cône d'angle $\pi/2$ du mouvement brownien limite. Cet énoncé implique un résultat de limite d'échelle pour la loi jointe des aires et des longueurs de bord des composantes connexes bornées du complémentaire des boucles FK sur la carte de volume infini. À la lumière du codage de Duplantier, Miller et Sheffield (2014), l'objet limite coïncide avec la loi jointe des aires et des longueurs de bords des composantes connexes bornées du complémentaire d'une collection de boucles CLE sur une surface indépendante dont la loi est donnée par la gravité quantique de Liouville.

MSC: Primary 60F17; 60G50; secondary 82B27

Keywords: Fortuin–Kasteleyn model; Random planar maps; Hamburger–cheeseburger bijection; Random walks in cones; Liouville quantum gravity; Schramm–Loewner evolution; Conformal loop ensembles; Peanosphere

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On the fourth moment condition for Rademacher chaos

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Abstract. Adapting the spectral viewpoint suggested in (*Ann. Probab.* **40** (6) (2012) 2439–2459) in the context of symmetric Markov diffusion generators and recently exploited in the non-diffusive setup of a Poisson random measure (*Ann. Probab.* (2017)), we investigate the fourth moment condition for discrete multiple integrals with respect to general, i.e. non-symmetric and non-homogeneous, Rademacher sequences and show that, in this situation, the fourth moment alone does not govern the asymptotic normality. Indeed, here one also has to take into consideration the maximal influence of the corresponding kernel functions. In particular, we show that there is *no* exact fourth moment theorem for discrete multiple integrals of order $m \geq 2$ with respect to a symmetric Rademacher sequence. This behavior, which is in contrast to the Gaussian (*Ann. Probab.* **33** (1) (2005) 177–193) and Poisson (*Ann. Probab.* (2017)) situation, closely resembles that of degenerate, non-symmetric U-statistics from the classical paper (*J. Multivariate Anal.* **34** (2) (1990) 275–289).

Résumé. En adaptant le point de vue spectral proposé par Ledoux (*Ann. Probab.* **40** (6) (2012) 2439–2459) dans le cadre des générateurs des diffusions Markoviennes, qui a également été exploité récemment dans la situation non-diffusive d'une mesure aléatoire de Poisson (*Ann. Probab.* (2017)), nous étudions la condition du quatrième moment pour des intégrales multiples discrètes relatives à des suites de Rademacher générales, c.à.d. non-symétriques et non-homogènes, et nous démontrons que, dans ce cas, le quatrième moment ne gouverne pas complètement leur normalité asymptotique. En effet, il faut aussi tenir compte de l'influence maximale des fonctions de noyau correspondantes. En particulier, nous démontrons qu'il n'y a pas de théorème du quatrième moment exact pour des intégrales multiples discrètes de l'ordre $m \geq 2$ relatives à une suite de Rademacher symétrique. Ce comportement, qui contraste avex les situations Gaussiennes (*Ann. Probab.* **33** (1) (2005) 177–193) et Poissoniennes (*Ann. Probab.* (2017)), ressemble fortement à celui des U-statistiques dégénerées et non-symétriques dans l'article classique (*J. Multivariate Anal.* **34** (2) (1990) 275–289).

MSC: 60F05; 60H07; 60H05

Keywords: Fourth moment theorem; Stein's method; Discrete Malliavin calculus; Rademacher sequences; Carré du champ operator

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Products of random matrices from polynomial ensembles

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Abstract. Very recently we have shown that the spherical transform is a convenient tool for studying the relation between the joint density of the singular values and that of the eigenvalues for bi-unitarily invariant random matrices. In the present work we discuss the implications of these results for products of random matrices. In particular, we derive a transformation formula for the joint densities of a product of two independent bi-unitarily invariant random matrices, the first from a polynomial ensemble and the second from a polynomial ensemble of derivative type. This allows us to re-derive and generalize a number of recent results in random matrix theory, including a transformation formula for the kernels of the corresponding determinantal point processes. Starting from these results, we construct a continuous family of random matrix ensembles interpolating between the products of different numbers of Ginibre matrices and inverse Ginibre matrices. Furthermore, we make contact to the asymptotic distribution of the Lyapunov exponents of the products of a large number of bi-unitarily invariant random matrices of fixed dimension.

Résumé. Très récemment nous avons montré que la transformée sphérique est un outil pratique pour étudier la relation entre la densité conjointe des valeurs singulières et celle des valeurs propres pour des matrices aléatoires bi-unitairement invariantes. Dans le travail présent, nous discutons les implications de ces résultats pour les produits de matrices aléatoires. En particulier, nous dérivons une formule de transformation pour les densités conjointes d'un produit de deux matrices aléatoires bi-unitairement invariantes indépendantes, la première d'un ensemble polynomial et la seconde d'un ensemble polynomial de type dérivé. Cela nous permet de redériver et de généraliser certains résultats récents dans la théorie des matrices aléatoires, y compris une formule de transformation pour les noyaux des processus ponctuels déterminantals associés. A partir de ces résultats, nous construisons une famille continue d'ensembles de matrices aléatoires interpolant entre les produits de différents nombres de matrices de Ginibre et de matrices de Ginibre inverses. De plus, nous établissons un lien avec la distribution asymptotique des exposants de Lyapunov des produits d'un grand nombre de matrices aléatoires bi-unitairement invariantes de dimension fixe.

MSC: Primary 60B20; secondary 15B52; 43A90; 60B15; 37H15

Keywords: Products of independent random matrices; Polynomial ensembles; Singular value distributions; Eigenvalue distributions; Spherical transform; Multiplicative convolution; Infinite divisibility; Lyapunov exponents; Stability exponents

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Barrier estimates for a critical Galton–Watson process and the cover time of the binary tree

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Abstract. For the critical Galton–Watson process with geometric offspring distributions we provide sharp barrier estimates for barriers which are (small) perturbations of linear barriers. These are useful in analyzing the cover time of finite graphs in the critical regime by random walk, and the Brownian cover times of compact two-dimensional manifolds. As an application of the barrier estimates, we prove that if C_L denotes the cover time of the binary tree of depth L by simple walk, then $\sqrt{C_L/2^{L+1}} - \sqrt{2\log 2}L + \log L/\sqrt{2\log 2}$ is tight. The latter improves results of Aldous (*J. Math. Anal. Appl.* **157** (1991) 271–283), Bramson and Zeitouni (*Ann. Probab.* **37** (2009) 615–653) and Ding and Zeitouni (*Stochastic Process. Appl.* **122** (2012) 2117–2133). In a subsequent article we use these barrier estimates to prove tightness of the Brownian cover time for compact two-dimensional manifolds.

Résumé. Pour le processus critique de Galton–Watson avec loi de reproduction géométrique de la progéniture, nous fournissons des estimations fines de barrière pour des obstacles qui sont des (petites) perturbations de barrières linéaires. Les estimations sont utiles pour analyser le temps de recouvrement, par une marche aleatoire, de graphes finis dans le régime critique, et les temps de recouvrement brownien de variétés bidemensionelles compactes. Comme application des estimations de barrière, nous prouvons que si C_L dénote le temps de recouvrement de l'arbre binaire de profondeur L par une marche aleatoire simple, la suite $\sqrt{C_L/2^{L+1}} - \sqrt{2\log 2}L + \log L/\sqrt{2\log 2}$ est tendue. Ce dernier resultat améliore les résultats d'Aldous (*J. Math. Anal. Appl.* **157** (1991) 271–283), Bramson et Zeitouni (*Ann. Probab.* **37** (2009) 615–653) et Ding et Zeitouni (*Stochastic Process. Appl.* **122** (2012) 2117–2133). Dans un article compagnon, nous utilisons ces estimations de barrière pour prouver la tension du temps de recouvrement brownien pour des variétés riemanniennes compactes en deux dimensions.

MSC: 60J80; 60J85; 60G50

Keywords: Galton–Watson process; Cover time; Binary tree; Barrier estimates

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Local limits of large Galton–Watson trees rerooted at a random vertex

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Abstract. We discuss various forms of convergence of the vicinity of a uniformly at random selected vertex in random simply generated trees, as the size tends to infinity. For the standard case of a critical Galton–Watson tree conditioned to be large the limit is the invariant random sin-tree constructed by Aldous (1991). In the condensation regime, we describe in complete generality the asymptotic local behaviour from a random vertex up to its first ancestor with large degree. Beyond this distinguished ancestor, different behaviour may occur, depending on the branching weights. In a subregime of complete condensation, we obtain convergence toward a novel limit tree, that describes the asymptotic shape of the vicinity of the full path from a random vertex to the root vertex. This includes the case where the offspring distribution follows a power law up to a factor that varies slowly at infinity.

Résumé. Nous discutons de plusieurs formes de convergence du voisinage d'un sommet aléatoire uniforme dans des arbres aléatoires simplement générés, lorsque leur taille tend vers l'infini. Pour le cas standard d'un arbre de Galton–Watson critique conditionné à être grand, la limite est le sin-tree invariant aléatoire construit par Aldous (1991). Dans le régime de condensation, nous décrivons en toute généralité le comportement asymptotique local depuis un sommet aléatoire jusqu'à son premier ancêtre de grand degré. Au delà de cet ancêtre distingué, différents comportements peuvent apparaître selon les poids de branchement. Dans un sous-régime de condensation complète, nous obtenons la convergence vers un nouvel arbre limite, qui décrit la forme asymptotique du voisinage du chemin complet depuis un sommet aléatoire jusqu'à la racine. Cela inclut le cas où la distribution de la descendance suit une loi de puissance, à un facteur près qui varie lentement à l'infini.

MSC: Primary 60J80; secondary 60B10

Keywords: Local weak limits; Simply generated trees; Fringe distributions

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Branching diffusion representation of semilinear PDEs and Monte Carlo approximation

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Abstract. We provide a representation result of parabolic semi-linear PDEs, with polynomial nonlinearity, by branching diffusion processes. We extend the classical representation for KPP equations, introduced by Skorokhod [*Theory Probab. Appl.* **9** (1964) 445–449], Watanabe [*J. Math. Kyoto Univ.* **4** (1965) 385–398] and McKean [*Comm. Pure Appl. Math.* **28** (1975) 323–331], by allowing for polynomial nonlinearity in the pair (u, Du), where u is the solution of the PDE with space gradient Du. Similar to the previous literature, our result requires a non-explosion condition which restrict to "small maturity" or "small nonlinearity" of the PDE. Our main ingredient is the Malliavin automatic differentiation technique as in [*Ann. Appl. Probab.* **27** (2017) 3305–3341], based on the Malliavin integration by parts, which allows to account for the nonlinearities in the gradient. As a consequence, the particles of our branching diffusion are marked by the nature of the nonlinearity. This new representation has very important numerical implications as it is suitable for Monte Carlo simulation. Indeed, this provides the first numerical method for high dimensional nonlinear PDEs with error estimate induced by the dimension-free central limit theorem. The complexity is also easily seen to be of the order of the squared dimension. The final section of this paper illustrates the efficiency of the algorithm by some high dimensional numerical experiments.

Résumé. Nous obtenons une représentation de la solution u d'une EDP semi-linéaire parabolique, avec une nonlinéarité polynomiale, par le biais d'un processus de diffusion branchant. Nous étendons ainsi le résultat de représentation classique pour les équations KPP, introduit par Skorokhod [*Theory Probab. Appl.* **9** (1964) 445–449], Watanabe [*J. Math. Kyoto Univ.* **4** (1965) 385–398] et McKean [*Comm. Pure Appl. Math.* **28** (1975) 323–331], au cas d'une nonlinéarité polynomiale en (u, Du). Bien évidemment, une telle non linéarité polynomiale requiert une condition de non explosion, qui est équivalent à une restriction de l'horizon, ou à une restriction de la taille de la perturbation nonlinéaire. L'ingrédient essentiel pour notre représentation est la technique de différentiation automatique de type Malliavin comme dans [*Ann. Appl. Probab.* **27** (2017) 3305–3341], qui permet de traiter la nonlinéarité en Du. Par conséquent, les particules de notre processus de branchement sont marquées par la nature de la nonlinéarité. Nous développons également une application importante de cette nouvelle représentation à l'approximation numérique de la solution d'une telle EDP par la méthode de Monte Carlo. Cette approximation est particulièrement intéressante en grande dimension du fait que l'estimation de l'erreur, induite par le théorème central limite, est indépendante de la dimension. La complexité de cet algorithme est de l'ordre du carré de la dimension. Dans le dernier paragraphe du papier, nous illustrons l'efficacité de cette méthode d'approximation numérique dans le cadre d'une équation de Burgers en dimension $d = 20$.

MSC: 60J85; 91G60

Keywords: Semilinear PDEs; Branching processes; Monte-Carlo methods

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Large deviations for the two-dimensional stochastic Navier–Stokes equation with vanishing noise correlation[1]

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Abstract. We are dealing with the validity of a large deviation principle for the two-dimensional Navier–Stokes equation, with periodic boundary conditions, perturbed by a Gaussian random forcing. We are here interested in the regime where both the strength of the noise and its correlation are vanishing, on a length scale ϵ and $\delta(\epsilon)$, respectively, with $0 < \epsilon, \delta(\epsilon) \ll 1$. Depending on the relationship between ϵ and $\delta(\epsilon)$ we will prove the validity of the large deviation principle in different functional spaces.

Résumé. Nous considérons les équations de Navier–Stokes avec conditions aux limites périodiques et perturbées par une force aléatoire gaussienne et démontrons un principe de grande déviation. Le régime étudié est celui-ci où l'amplitude du bruit et sa corrélation tendent vers zéro aux vitesses ϵ et $\delta(\epsilon)$, avec $0 < \epsilon, \delta(\epsilon) \ll 1$. Le principe de grande déviation est démontré dans différent espaces fonctionnels selon le comportement $\delta(\epsilon)$ en fonction de ϵ.

MSC: 60H15; 60F10; 35Q30

Keywords: Stochastic Navier–Stokes equation; Large deviations; Weak convergence approach to large deviations; Rough noise

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Brownian disks and the Brownian snake

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Abstract. We provide a new construction of the Brownian disks, which have been defined by Bettinelli and Miermont as scaling limits of quadrangulations with a boundary when the boundary size tends to infinity. Our method is very similar to the construction of the Brownian map, but it makes use of the positive excursion measure of the Brownian snake which has been introduced recently. This excursion measure involves a continuous random tree whose vertices are assigned nonnegative labels, which correspond to distances from the boundary in our approach to the Brownian disk. We provide several applications of our construction. In particular, we prove that the uniform measure on the boundary can be obtained as the limit of the suitably normalized volume measure on a small tubular neighborhood of the boundary. We also prove that connected components of the complement of the Brownian net are Brownian disks, as it was suggested in the recent work of Miller and Sheffield. Finally, we show that connected components of the complement of balls centered at the distinguished point of the Brownian map are independent Brownian disks, conditionally on their volumes and perimeters.

Résumé. Nous donnons une nouvelle construction des disques browniens, qui ont été définis par Bettinelli et Miermont comme limites d'échelle de quadrangulations avec frontière quand la taille de la frontière tend vers l'infini. Notre méthode est semblable à la construction de la carte brownienne, mais elle utilise la mesure d'excursion positive du serpent brownien introduite récemment. Cette mesure d'excursion implique un arbre aléatoire continu dont les sommets reçoivent des labels positifs, qui correspondent aux distances depuis la frontière dans notre approche du disque brownien. Nous donnons plusisurs applications de cette construction. En particulier, nous montrons que la mesure uniforme sur la frontière peut être obtenue comme limite de la mesure de volume (convenablement normalisée) sur un petit voisinage tubulaire de la frontière. Nous montrons aussi que les composantes connexes du complémentaire du filet brownien sont des disques browniens, comme cela est suggéré dans le travail récent de Miller et Sheffield. Finalement, nous montrons que les composantes connexes du complémentaire d'une boule centrée au point distingué de la carte brownienne sont, conditionnellement à leurs volumes et leurs périmètres, des disques browniens indépendants.

MSC: 60D05; 60C05

Keywords: Brownian disk; Brownian map; Excursion measure; Brownian snake; Uniform measure on boundary; Continuous random tree; Connected components of complement of balls

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Conditioning a Brownian loop-soup cluster on a portion of its boundary

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Abstract. We show that if one conditions a cluster in a Brownian loop-soup L (of any intensity) in a two-dimensional domain by a portion ∂ of its outer boundary, then in the remaining domain, the union of all the loops of L that touch ∂ satisfies the conformal restriction property while the other loops in L form an independent loop-soup. This result holds when one discovers ∂ in a natural Markovian way, such as in the exploration procedures that have been defined in order to actually construct the Conformal Loop Ensembles as outer boundaries of loop-soup clusters. This result implies among other things that a phase transition occurs at $c = 14/15$ for the connectedness of the loops that touch ∂.

Our results can be viewed as an extension of some of the results in our paper (*J. Eur. Math. Soc.* (2019) to appear) in the following two directions: There, a loop-soup cluster was conditioned on its entire outer boundary while we discover here only part of this boundary. And, while it was explained in (*J. Eur. Math. Soc.* (2019) to appear) that the strong decomposition using a Poisson point process of excursions that we derived there should be specific to the case of the critical loop-soup, we show here that in the subcritical cases, a weaker property involving the conformal restriction property nevertheless holds.

Résumé. Dans le présent article, nous étudions certaines propriétés des amas de lacets browniens, dans une soupe de lacets browniens d'intensité c dans un domaine du plan, pour toute intensité $c \leq 1$.

Notre principal résultat dit que si l'on découvre de manière Markovienne une portion ∂ du bord extérieur d'un tel amas, alors dans le domaine restant, la loi conditionnelle de l'union de tous les lacets dans L qui touchent ∂ satisfait la propriété de restriction conforme tandis que les autres lacets dans L forment une soupe de lacets indépendante. Ceci implique en particulier l'existence d'une transition de phase à $c = 14/15$ pour la connectivité de l'ensemble des lacets qui touchent ∂.

Nos résultats constituent une extension de certains résultats de notre papier (*J. Eur. Math. Soc.* (2019) to appear) dans les deux directions suivantes: Dans (*J. Eur. Math. Soc.* (2019) to appear), un cluster de lacets est conditionné par son bord extérieur entier tandis que nous découvrons ici seulement une partie de ce bord. En outre, dans (*J. Eur. Math. Soc.* (2019) to appear), nous expliquons que la description que nous donnons de l'ensemble des lacets qui touchent ce bord via un processus ponctuel de Poisson d'excursions est spécifique au cas de la soupe de lacets critique ($c = 1$), nous montrons ici que dans les cas sous-critiques $c < 1$, une propriété plus faible de restriction conforme reste néanmoins vraie.

MSC: 60J65; 60J67; 60K35

Keywords: Brownian loop-soups; Conformal loop ensembles; Schramm–Loewner evolution; Conformal restriction

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Intertwinings and Stein's magic factors for birth–death processes

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Abstract. This article investigates second order intertwinings between semigroups of birth–death processes and discrete gradients on $\mathbb{N}$. It goes one step beyond a recent work of Chafaï and Joulin which establishes and applies to the analysis of birth–death semigroups a first order intertwining. Similarly to the first order relation, the second order intertwining involves birth–death and Feynman–Kac semigroups and weighted gradients on $\mathbb{N}$, and can be seen as a second derivative relation. As our main application, we provide new quantitative bounds on the Stein factors of discrete distributions. To illustrate the relevance of this approach, we also derive approximation results for the mixture of Poisson and geometric laws.

Résumé. Cet article établit l'existence d'entrelacements au second ordre entre semi-groupes relatifs aux processus de naissance-mort et gradients discrets sur $\mathbb{N}$, allant ainsi un pas plus loin que les travaux récents de Chafaï et Joulin, qui concernent les entrelacements au premier ordre et leur application à l'analyse des semi-groupes de naissance-mort. Comme la relation du premier ordre, l'entrelacement de second ordre fait intervenir des semi-groupes de naissance-mort et de Feynman–Kac et des gradients à poids sur $\mathbb{N}$, et peut s'interpréter comme une relation de dérivation à l'ordre deux. Comme application principale, nous établissons des nouvelles bornes sur les facteurs de Stein relatifs aux distributions discrètes, et nous donnons également des résultats d'approximation pour le mélange de lois géométriques et le mélange de lois de Poisson, qui illustrent la pertinence de notre approche.

MSC: Primary 60E15; secondary 60J80; 47D08; 60E05; 60F05

Keywords: Birth–death processes; Feynman–Kac semigroups; Intertwinings; Stein's factors; Stein's method; Distances between probability distributions

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Mixing and decorrelation in infinite measure: The case of the periodic Sinai billiard

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Abstract. We investigate the question of the rate of mixing for observables of a $\mathbb{Z}^d$-extension of a probability preserving dynamical system with good spectral properties. We state general mixing results, including expansions of every order. The main motivation of this article is the study of mixing rates for smooth observables of the $\mathbb{Z}^2$-periodic Sinai billiard, with different kinds of results depending on whether the horizon is finite or infinite. We establish a first order mixing result when the horizon is infinite. In the finite horizon case, we establish an asymptotic expansion of every order, enabling the study of the mixing rate even for observables with null integrals. This result is related to an Edgeworth expansion in the local limit theorem.

Résumé. Cet article est une contribution à l'étude du mélange d'observables de systèmes dynamiques préservant une mesure infinie. Nous étudions le cas de $\mathbb{Z}^d$-extensions de systèmes dynamiques probabilisés ayant de bonnes propriétés spectrales. Nous établissons des résultats généraux et les illustrons par plusieurs exemples. Notre motivation principale est l'étude de la vitesse de mélange pour des observables régulières du billard de Sinai $\mathbb{Z}^2$-périodique, pour lequel nous obtenons des résultats de types différents selon que l'horizon soit fini ou infini. Nous établissons un résultat de mélange du premier ordre lorsque l'horizon est infini. Dans le cas où l'horizon est fini, nous établissons un développement asymptotique de tout ordre, permettant l'étude de la vitesse de mélange pour des observables d'intégrale nulle. Ce dernier résultat est relié à un développement de Edgeworth dans le théorème limite local.

MSC: 37A25

Keywords: Sinai; Billiard; Lorentz process; Young tower; Local limit theorem; Decorrelation; Mixing; Infinite measure; Edgeworth expansion

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The local limit of random sorting networks

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Abstract. A sorting network is a geodesic path from $12\cdots n$ to $n\cdots 21$ in the Cayley graph of S_n generated by adjacent transpositions. For a uniformly random sorting network, we establish the existence of a local limit of the process of space–time locations of transpositions in a neighbourhood of an for $a \in [0,1]$ as $n \to \infty$. Here time is scaled by a factor of $1/n$ and space is not scaled.

The limit is a swap process U on $\mathbb{Z}$. We show that U is stationary and mixing with respect to the spatial shift and has time-stationary increments. Moreover, the only dependence on a is through time scaling by a factor of $\sqrt{a(1-a)}$.

To establish the existence of U, we find a local limit for staircase-shaped Young tableaux. These Young tableaux are related to sorting networks through a bijection of Edelman and Greene.

Résumé. Un réseau de tri est un chemin géodésique de $12\cdots n$ à $n\cdots 21$ dans le graphe de Cayley de S_n généré par les transpositions adjacentes. Pour un réseau de tri uniforme, on établit l'existence d'une limite locale du processus des positions espace-temps des transpositions dans un voisinage de an pour $a \in [0,1]$ lorsque $n \to \infty$. Ici, le temps est mis à l'échelle par un facteur de $1/n$ et l'espace n'est pas mis à l'échelle.

La limite est un processus d'échange U sur $\mathbb{Z}$. On montre que U est stationnaire et mélangeant par rapport au déplacement spatial, et qu'il a des incréments de temps qui sont stationnaires. De plus, la seule dépendance sur a est à travers une mise à l'échelle temporelle par un facteur de $\sqrt{a(1-a)}$.

Pour établir l'existence de U, on trouve une limite locale pour les tableaux de Young en forme d'escalier. Ces tableaux de Young sont reliés aux réseaux de tri à travers une bijection d'Edelman et Greene.

MSC: 60C05; 05E10; 68P10

Keywords: Sorting network; Random sorting network; Reduced decomposition; Young tableau; Local limit

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Finite rank perturbations in products of coupled random matrices: From one correlated to two Wishart ensembles

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Abstract. We compare finite rank perturbations of the following three ensembles of complex rectangular random matrices: First, a generalised Wishart ensemble with one random and two fixed correlation matrices introduced by Borodin and Péché, second, the product of two independent random matrices where one has correlated entries, and third, the case when the two random matrices become also coupled through a fixed matrix. The singular value statistics of all three ensembles is shown to be determinantal and we derive double contour integral representations for their respective kernels. Three different kernels are found in the limit of infinite matrix dimension at the origin of the spectrum. They depend on finite rank perturbations of the correlation and coupling matrices and are shown to be integrable. The first kernel (I) is found for two independent matrices from the second, and two weakly coupled matrices from the third ensemble. It generalises the Meijer G-kernel for two independent and uncorrelated matrices. The third kernel (III) is obtained for the generalised Wishart ensemble and for two strongly coupled matrices. It further generalises the perturbed Bessel kernel of Desrosiers and Forrester. Finally, kernel (II), found for the ensemble of two coupled matrices, provides an interpolation between the kernels (I) and (III), generalising previous findings of part of the authors.

Résumé. Les perturbations de rang fini des trois ensembles de matrices aléatoires complexes rectangulaires suivants sont comparés: d'abord un ensemble de Wishart généralisé, avec une matrice aléatoire et deux matrices de corrélation fixées, introduit par Borodin et Péché ; ensuite le produit de deux matrices aléatoires indépendantes, dont une a des éléments corrélés ; enfin le cas où deux matrices aléatoires sont couplées par une matrice fixée. Nous prouvons que la statistique des valeurs singulières des trois ensembles est déterminantale et nous dérivons des représentations en termes d'intégrales de contour doubles pour leurs noyaux respectifs. Dans la limite de dimension de matrice infinie à l'origine du spectre, on trouve trois noyaux différents, qui dépendent de la perturbation du rang fini des matrices de corrélation et du couplage et s'avèrent être intégrables. Le premier noyau (I) est trouvé pour le cas de deux matrices indépendantes du second ensemble, et pour celui de deux matrices faiblement couplées du troisième ensemble. Ce noyau généralise celui du type Meijer-G, valable pour deux matrices indépendantes et non corrélées. Le troisième noyau (III) est obtenu pour l'ensemble de Wishart généralisé et pour deux matrices couplées de façon forte. Celui-là généralise le noyau de Bessel perturbé de Desrosiers et Forrester. Finalement, le noyau (II), qui est trouvé pour l'ensemble de deux matrices couplées, représente une interpolation entre les noyaux (I) et (III), ce qui généralise des résultats précédemment obtenus par certains des auteurs.

MSC: 15A52; 60G55; 33C45

Keywords: Products of random matrices; Correlated Wishart ensemble; Determinantal point processes; Biorthogonal ensembles; Finite rank perturbations

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Functional limit theorem for the self-intersection local time of the fractional Brownian motion

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Abstract. Let $\{B_t\}_{t\geq 0}$ be a d-dimensional fractional Brownian motion with Hurst parameter $0 < H < 1$, where $d \geq 2$. Consider the approximation of the self-intersection local time of B, defined as

$$I_T^{\varepsilon} = \int_0^T \int_0^t p_{\varepsilon}(B_t - B_s)\, ds\, dt,$$

where $p_{\varepsilon}(x)$ is the heat kernel. We prove that the process $\{I_T^{\varepsilon} - \mathbb{E}[I_T^{\varepsilon}]\}_{T\geq 0}$, rescaled by a suitable normalization, converges in law to a constant multiple of a standard Brownian motion for $\frac{3}{2d} < H \leq \frac{3}{4}$ and to a multiple of a sum of independent Hermite processes for $\frac{3}{4} < H < 1$, in the space $C[0,\infty)$, endowed with the topology of uniform convergence on compacts.

Résumé. Soit $\{B_t\}_{t\geq 0}$ un mouvement brownien fractionnaire d-dimensionel avec paramètre de Hurst $0 < H < 1$, où $d \geq 2$. On considère l'approximation du temps local d'auto-intersection du processus B, défini comme

$$I_T^{\varepsilon} = \int_0^T \int_0^t p_{\varepsilon}(B_t - B_s)\, ds\, dt,$$

où $p_{\varepsilon}(x)$ est le noyau de la chaleur. Nous démontrons que le processus $\{I_T^{\varepsilon} - \mathbb{E}[I_T^{\varepsilon}]\}_{T\geq 0}$, rééchelonné avec une normalisation convenable, converge en loi vers un mouvement brownien multiplié par une constante si $\frac{3}{2d} < H \leq \frac{3}{4}$ et vers une somme de processus de Hermite indépendants multipliée par une constante si $\frac{3}{4} < H < 1$, dans l'espace $C[0,\infty)$, muni de la topologie de la convergence uniforme sur les compacts.

MSC: 60G05; 60H07; 60G15; 60F17

Keywords: Fractional Brownian motion; self-intersection local time; Wiener chaos expansion; central limit theorem

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Universality of Ghirlanda–Guerra identities and spin distributions in mixed p-spin models

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Abstract. We prove universality of the Ghirlanda–Guerra identities and spin distributions in the mixed p-spin models. The assumption for the universality of the identities requires exactly that the coupling constants have zero means and finite variances, and the result applies to the Sherrington–Kirkpatrick model. As an application, we obtain weakly convergent universality of spin distributions in the generic p-spin models under the condition of two matching moments. In particular, certain identities for 3-overlaps and 4-overlaps under the Gaussian disorder follow. Under the stronger mode of total variation convergence, we find that universality of spin distributions in the mixed p-spin models holds if mild dilution of connectivity by the Viana–Bray diluted spin glass Hamiltonians is present and the first three moments of coupling constants in the mixed p-spin Hamiltonians match. These universality results are in stark contrast to the characterization of spin distributions in the undiluted mixed p-spin models, which is known up to now that four matching moments are required in general.

Résumé. Nous prouvons l'universalité des identités de Ghirlanda–Guerra et des distributions de spin dans les mélanges de modèles à p-spin. L'hypothèse de l'universalité des identités exige précisément que les constantes de couplage aient des moyennes nulles et des variances finies; le résultat s'applique au modèle de Sherrington–Kirkpatrick. Comme application, nous obtenons une universalité faiblement convergente des distributions de spin dans les modèles à p-spin génériques à condition d'avoir deux moments analogues. En particulier, certaines identités pour 3 chevauchements et 4 chevauchements selon le modèle de perturbation gaussienne s'ensuivent. Sous le mode plus fort de la convergence de la variation totale, nous trouvons que l'universalité des distributions de spin dans les mélanges de modèles à p-spin est maintenue si une légère dilution de la connectivité par les Hamiltoniens de verre de spin du modèle dilué de Viana–Bray est présente et que les trois premiers moments des constantes de couplage dans les Hamiltoniens des mélanges de modèles à p-spin mixtes concordent. Ces résultats d'universalité s'opposent fortement à la caractérisation des distributions de spin dans les mélanges de modèles à p-spin non dilués, qui sont connus jusqu'à présent pour avoir besoin en général de quatre moments analogues.

MSC: 82D30; 60K35; 82B44

Keywords: Mixed p-spin models; The Ghirlanda–Guerra identities; Universality; Ultrametricity; The Viana–Bray diluted spin glass model

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Convergence of the free Boltzmann quadrangulation with simple boundary to the Brownian disk

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Abstract. We prove that the free Boltzmann quadrangulation with simple boundary and fixed perimeter, equipped with its graph metric, natural area measure, and the path which traces its boundary converges in the scaling limit to the free Boltzmann Brownian disk. The topology of convergence is the so-called Gromov–Hausdorff–Prokhorov-uniform (GHPU) topology, the natural analog of the Gromov–Hausdorff topology for curve-decorated metric measure spaces. From this we deduce that a random quadrangulation of the sphere decorated by a $2l$-step self-avoiding loop converges in law in the GHPU topology to the random curve-decorated metric measure space obtained by gluing together two independent Brownian disks along their boundaries.

Résumé. Nous démontrons que la quadrangulation de Boltzmann libre avec un bord simple de périmètre fixé, munie de sa métrique de graphe, de sa mesure d'aire naturelle, et du chemin qui décrit sa frontière, converge dans la limite d'échelle vers le disque brownien libre de Boltzmann. La topologie de cette convergence est celle de Gromov–Hausdorff–Prokhorov-uniforme (GHPU), qui est l'analogue naturel de la topologie de Gromov-Hausdroff pour des espaces métriques mesurés décorés par une courbe. Nous déduisons de cela qu'une quadrangulation aléatoire de la sphère, décorée par une marche aléatoire auto-évitante de longueur $2l$, converge en loi pour la topologie GHPU vers l'espace métrique mesuré et décoré par une courbe que l'on obtient en recollant ensemble deux disques browniens indépendants le long de leurs bords.

MSC: 60D05; 60F17; 05C80

Keywords: Random planar maps; Brownian map; Brownian disk; Quadrangulation with simple boundary; Self-avoiding walk; Gromov–Hausdorff–Prokhorov-uniform topology

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Ergodicity of a system of interacting random walks with asymmetric interaction

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Abstract. We study N interacting random walks on the positive integers. Each particle has drift δ towards infinity, a reflection at the origin, and a drift towards particles with lower positions. This inhomogeneous mean field system is shown to be ergodic only when the interaction is strong enough. We focus on this latter regime, and point out the effect of piles of particles, a phenomenon absent in models of interacting diffusion in continuous space.

Résumé. Nous étudions N marches aléatoires interagissantes sur les entiers naturels. Chaque particule a une dérive δ vers l'infini, une réflexion à l'origine, ainsi qu'une dérive vers les particules de positions plus petites. Nous montrons que ce système de champ moyen inhomogène est ergodique lorsque l'interaction est assez forte. Nous nous concentrons sur ce dernier régime, et mettons en lumière le rôle des empilements de particules sur un même site, phénomène absent dans les modèles continus.

MSC: 60K35; 82C44

Keywords: Interacting particle systems; Mean-field interaction; Non-reversibility

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