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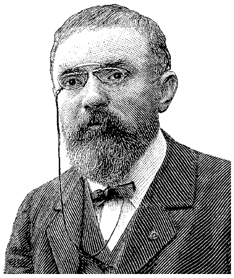

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The velocity of 1d Mott variable-range hopping with external field

Alessandra Faggionato[a], Nina Gantert[b] and Michele Salvi[c]

[a]*Dipartimento di Matematica, Università di Roma "La Sapienza", P.le Aldo Moro 2, 00185 Roma, Italy. E-mail: faggiona@mat.uniroma1.it*
[b]*Fakultät für Mathematik, Technische Universität München, 85748 Garching, Germany. E-mail: gantert@ma.tum.de*
[c]*Université Paris-Dauphine, PSL Research University, CNRS, [UMR 7534], CEREMADE, 75016 Paris, France. E-mail: salvi@ceremade.dauphine.fr*

Abstract. Mott variable-range hopping is a fundamental mechanism for low-temperature electron conduction in disordered solids in the regime of Anderson localization. In a mean field approximation, it reduces to a random walk (shortly, Mott random walk) on a random marked point process with possible long-range jumps.

We consider here the one-dimensional Mott random walk and we add an external field (or a bias to the right). We show that the bias makes the walk transient, and investigate its linear speed. Our main results are conditions for ballisticity (positive linear speed) and for sub-ballisticity (zero linear speed), and the existence in the ballistic regime of an invariant distribution for the environment viewed from the walker, which is mutually absolutely continuous with respect to the original law of the environment. If the point process is a renewal process, the aforementioned conditions result in a sharp criterion for ballisticity. Interestingly, the speed is not always continuous as a function of the bias.

Résumé. Le « Mott variable-range hopping » est un mécanisme décrivant la conduction des electrons dans des solides désordonnés dans le régime de localisation d'Anderson. Sous l'approximation de champ moyen, le modèle se réduit à une marche aléatoire (marche aléatoire de Mott) sur un processus ponctuel. Cette marche peut sauter d'un point du processus ponctuel à n'importe quel autre, les sauts ne sont donc pas limités en taille.

Nous considerons une marche aléatoire de Mott unidimensionelle soumis à un champ extérieur (équivalent à un biais à droite). Nous montrons que la marche biaisée est transiente, et nous étudions sa vitesse linéaire. Nos résultats principaux sont des conditions pour la ballisticité (vitesse strictement positif) et la sous-ballisticité (vitesse nulle). Dans le regime ballistique, nous montrons l'existence d'une mesure invariante pour l'environment vu par la particule, absolument continue par rapport à la mesure originale. Si le processus ponctuel est un processus de renouvellement, nos conditions deviennent une condition nécessaire et suffisante pour la ballisticité. Nous montrons ainsi que la vitesse de la marche n'est pas, en général, une fonction continue du biais.

MSC: 60K37; 82D30; 60G50; 60G55

Keywords: Random walk in random environment; Disordered media; Ballisticity; Environment viewed from the walker; Electron transport in disordered solids

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Spectral gap for the stochastic quantization equation on the 2-dimensional torus

Pavlos Tsatsoulis and Hendrik Weber

University of Warwick, Coventry, UK. E-mail: p.tsatsoulis@warwick.ac.uk; hendrik.weber@warwick.ac.uk

Abstract. We study the long time behavior of the stochastic quantization equation. Extending recent results by Mourrat and Weber (Global well-posedness of the dynamic ϕ^4 in the plane (2015) Preprint) we first establish a strong non-linear dissipative bound that gives control of moments of solutions at all positive times independent of the initial datum. We then establish that solutions give rise to a Markov process whose transition semigroup satisfies the strong Feller property. Following arguments by Chouk and Friz (Support theorem for a singular SPDE: the case of gPAM (2016) Preprint) we also prove a support theorem for the laws of the solutions. Finally all of these results are combined to show that the transition semigroup satisfies the Doeblin criterion which implies exponential convergence to equilibrium.

Along the way we give a simple direct proof of the Markov property of solutions and an independent argument for the existence of an invariant measure using the Krylov–Bogoliubov existence theorem. Our method makes no use of the reversibility of the dynamics or the explicit knowledge of the invariant measure and it is therefore in principle applicable to situations where these are not available, e.g. the vector-valued case.

Résumé. Nous étudions le comportement sur le long terme de l'équation de quantification stochastique. Dans la continuité de récents résultats par Mourrat et Weber (Global well-posedness of the dynamic ϕ^4 in the plane (2015) Preprint), nous établissons en premier lieu une borne dissipative forte non-linéaire qui contrôle les moments des solutions, pour tout choix de temps, indépendamment des conditions initiales. Nous prouvons ensuite que les solutions génèrent un processus Markovien dont le semigroupe satisfait la propriété de Feller forte. Nous obtenons également un théorème pour le support des lois des solutions grâce à des arguments adaptés de Chouk et Friz (Support theorem for a singular SPDE: the case of gPAM (2016) Preprint). Enfin, en combinant tous ces résultats, nous montrons que le semigroupe de transition satisfait le critère de Doeblin, ce qui entraine une convergence exponentielle vers l'équilibre.

Nous obtenons également au passage une preuve directe de la propriété de Markov pour les solutions, ainsi qu'un argument indépendant pour l'existence de mesures invariantes en utilisant le théorème d'existence de Krylov–Bogoliubov. Notre méthode n'utilise pas le caractère réversible de la dynamique ni la connaissance explicite de la mesure invariante, et peut donc en théorie s'appliquer dans des cas où ces propriétés ne sont pas connues, par exemple le cas vectoriel.

MSC: 37A25; 60H15; 81T08

Keywords: Singular SPDEs; Strong Feller property; Support theorem; Exponential mixing

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Asymptotics of random domino tilings of rectangular Aztec diamonds

Alexey Bufetov and Alisa Knizel

Department of Mathematics, Massachusetts Institute of Technology, Cambridge, MA, USA.
E-mail: alexey.bufetov@gmail.com; alisik@math.mit.edu

Abstract. We consider asymptotics of a domino tiling model on a class of domains which we call rectangular Aztec diamonds. We prove the Law of Large Numbers for the corresponding height functions and provide explicit formulas for the limit. For a special class of examples, the explicit parametrization of the frozen boundary is given. It turns out to be an algebraic curve with very special properties. Moreover, we establish the convergence of the fluctuations of the height functions to the Gaussian Free Field in appropriate coordinates. Our main tool is a recently developed moment method for discrete particle systems.

Résumé. Nous nous intéressons aux propriétés asymptotiques d'un modèle de pavage par dominos sur une classe de domaines que nous appelons les diamants aztèques rectangulaires. Nous prouvons une loi des grands nombres pour les fonctions de hauteur correspondantes, et donnons des formules explicites pour la limite. Pour une classe d'exemples particulière, nous pouvons donner la paramétrisation explicite de la frontière gelée. Cette dernière se trouve être une courbe algébrique aux propriétés très particulières. De plus, nous établissons la convergence des fluctuations des fonctions de hauteur vers le champ libre gaussien dans des coordonnées appropriées. Notre outil principal est une méthode de moments développée récemment dans le cadre des systèmes de particules discrets.

MSC: Primary 60K35; secondary 60C05

Keywords: Random domino tilings; Central limit theorem; Moment method

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Scaling limit and ageing for branching random walk in Pareto environment

Marcel Ortgiese and Matthew I. Roberts

Department of Mathematical Sciences, University of Bath, Bath BA2 7AY, UK. E-mail: m.ortgiese@bath.ac.uk; mattiroberts@gmail.com

Abstract. We consider a branching random walk on the lattice, where the branching rates are given by an i.i.d. Pareto random potential. We show that the system of particles, rescaled in an appropriate way, converges in distribution to a scaling limit that is interesting in its own right. We describe the limit object as a growing collection of "lilypads" built on a Poisson point process in $\mathbb{R}^d$. As an application of our main theorem, we show that the maximizer of the system displays the ageing property.

Résumé. Nous considérons une marche aléatoire branchante sur un réseau, où les taux de branchement sont donnés par un potentiel aléatoire i.i.d. suivant des lois de Pareto. Nous montrons que le système de particules, renormalisé d'une façon idoine, converge en loi vers une limite d'échelle intéressante en elle-même. Nous décrivons l'objet limite comme une collection croissante de « nénuphars » construits à partir d'un processus de Poisson dans $\mathbb{R}^d$. Comme application de notre théorème principal, nous montrons que le maximiseur du système possède la propriété de vieillissement.

MSC: Primary 60K37; secondary 60J80

Keywords: Branching random walk; Random environment; Parabolic Anderson model; Intermittency

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The strong Feller property for singular stochastic PDEs

M. Hairer[a] and J. Mattingly[b]

[a]*Mathematics Research Centre, University of Warwick, UK. E-mail: m.hairer@warwick.ac.uk*
[b]*Departments of Mathematics and Statistical Science, Duke University, USA. E-mail: jonm@math.duke.edu*

Abstract. We show that the Markov semigroups generated by a large class of singular stochastic PDEs satisfy the strong Feller property. These include for example the KPZ equation and the dynamical Φ^4_3 model. As a corollary, we prove that the Brownian bridge measure is the unique invariant measure for the KPZ equation with periodic boundary conditions.

Résumé. Nous montrons que les semi-groupes de Markov engendrés par une classe large d'EDPs stochastiques singulières satisfont la propriété forte de Feller. Cette classe inclut par exemple l'équation KPZ et le modèle Φ^4_3. Nous montrons comme corollaire que la distribution du pont Brownien est l'unique mesure invariante pour l'équation KPZ avec conditions frontières périodiques.

MSC: 60H15; 37L55; 81S20

Keywords: Strong Feller; Random dynamical systems; Rough stochastic PDEs; Ergodicity; Stochastic quantisation; Girsanov

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Biased random walks on the interlacement set[1]

Alexander Fribergh[a,2] and Serguei Popov[b,3]

[a]Université de Montréal, Department of Mathematics and Statistics, Pavillon André-Aisenstadt 2920, chemin de la Tour, Montréal H3T 1J4, Canada. E-mail: fribergh@dms.umontreal.ca
[b]Department of Statistics, Institute of Mathematics, Statistics and Scientific Computation, University of Campinas – UNICAMP, rua Sérgio Buarque de Holanda 651, 13083–859, Campinas SP, Brazil. E-mail: popov@ime.unicamp.br

Abstract. We study a biased random walk on the interlacement set of $\mathbb{Z}^d$ for $d \geq 3$. Although the walk is always transient, we can show, in the case $d = 3$, that for any value of the bias the walk has a zero limiting speed and actually moves slower than any power.

Résumé. Nous étudions la marche biaisée sur un entrelac aléatoire de $\mathbb{Z}^d$ avec $d \geq 3$. Nous montrons que la marche est transiente mais que, dans le cas $d = 3$, elle est sous-ballistique pour toutes les valeurs du biais et que ses déplacements sont inférieurs à n'importe quel polynôme.

MSC: Primary 60K37; secondary 60G50; 82C41

Keywords: Random walk in random environment; Interlacement set

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Markov processes on the duals to infinite-dimensional classical Lie groups

Cesar Cuenca

Department of Mathematics, MIT, Cambridge, MA, USA. E-mail: cuenca@mit.edu

Abstract. We construct a four parameter z, z', a, b family of Markov dynamics that preserve the z-measures on the boundary of the branching graph for classical Lie groups of type B, C, D. Our guiding principle is the "method of intertwiners" used previously in [*J. Funct. Anal.* **263** (2012) 248–303] to construct Markov processes that preserve the zw-measures.

Résumé. Nous construisons une famille à quatre paramètres, z, z', a, b, de dynamiques de Markov qui préservent les z-mesures sur la frontière du graphe branchant pour les groupes de Lie classiques du type B, C, D. L'idée maîtresse est la « méthode des entrelacements » utilisée précédemment dans [*J. Funct. Anal.* **263** (2012) 248–303] pour construire un processus de Markov qui préserve les zw-mesures.

MSC: 60C05; 60J25; 60B15

Keywords: BC type z-measures; Infinite dimensional spaces; Intertwining processes; Doob h-transform

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Weak convergence of obliquely reflected diffusions

Andrey Sarantsev

Department of Statistics and Applied Probability, UC Santa Barbara. E-mail: sarantsev@pstat.ucsb.edu

Abstract. Burdzy and Chen (*Electron. J. Probab.* **3** (1998) 29–33) proved results on weak convergence of multidimensional normally reflected Brownian motions. We generalize their work by considering obliquely reflected diffusion processes. We require weak convergence of domains, which is stronger than convergence in Wijsman topology, but weaker than convergence in Hausdorff topology.

Résumé. Burdzy et Chen (*Electron. J. Probab.* **3** (1998) 29–33) ont montré des résultats portant sur la convergence faible des mouvements Browniens multidimensionnels avec réflexion normale. Nous généralisons leurs travaux dans le cas de processus de diffusion avec réflexion oblique. Notre résultat requiert la faible convergence des domaines. Notons que cette convergence est plus forte que la convergence dans la topologie de Wijsman, mais plus faible que celle de la topologie de Hausdorff.

MSC: Primary 60J60; secondary 60J55; 60J65; 60H10

Keywords: Reflected diffusions; Oblique reflection; Hausdorff convergence; Wijsman convergence; Weak convergence

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Fluctuations of bridges, reciprocal characteristics and concentration of measure

Giovanni Conforti

Department of Mathematics, Universität Leipzig, Augustusplatz 10, 04109 Leipzig, Germany. E-mail: giovanniconfort@gmail.com

Abstract. Conditions on the generator of a Markov process to control the fluctuations of its bridges are found. In particular, continuous time random walks on graphs and gradient diffusions are considered. Under these conditions, a concentration of measure inequality for the marginals of the bridge of a gradient diffusion and refined large deviation expansions for the tails of a random walk on a graph are derived. In contrast with the existing literature about bridges, all the estimates we obtain hold for non asymptotic time scales. New concentration of measure inequalities for pinned Poisson random vectors are also established. The quantities expressing our conditions are the so called *reciprocal characteristics* associated with the Markov generator.

Résumé. Dans cet article nous exhibons des conditions sur le générateur d'un processus de Markov qui nous permettent de quantifier les fluctuations de ses ponts. Nous nous intéressons plus précisément aux marches aléatoires sur les graphes et aux diffusions de type gradient. Nous démontrons une inégalité de concentration pour la loi marginale du pont d'une diffusion gradient ainsi qu'un principe de grandes déviations pour les queues d'une marche aléatoire sur un graphe. L'originalité de nos résultats réside dans le fait qu'ils sont valables pour toute échelle de temps, tandis que ceux qui préexistent dans la litérature sont uniquement asymptotiques. Nous établissons aussi des inégalités de concentration pour des vecteurs aléatoires poissoniens conditionnés. Les paramètres, dérivés des processus markoviens, qui interviennent dans les conditions mises en évidence, sont leurs invariants réciproques.

MSC: 60J27; 60J75

Keywords: Bridges; Concentration of measure; Reciprocal characteristics; Tail asymptotic

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Construction of Malliavin differentiable strong solutions of SDEs under an integrability condition on the drift without the Yamada–Watanabe principle

David R. Baños[a], Sindre Duedahl[a], Thilo Meyer-Brandis[b] and Frank Proske[a]

[a]*University of Oslo, Moltke Moes vei 35 Niels Henrik Abels hus 0851 Oslo, Norway.*
E-mail: davidru@math.uio.no; sindred@math.uio.no; proske@math.uio.no
[b]*LMU University of Munich, Mathematics Institute, Theresienstr. 39, D-80333 Munich, Germany. E-mail: Meyer-Brandis@math.lmu.de*

Abstract. In this paper we aim at employing a compactness criterion of Da Prato, Malliavin, Nualart (C. R. Math. Acad. Sci. Paris **315** (1992) 1287–1291) for square integrable Brownian functionals to construct strong solutions of SDE's under an integrability condition on the drift coefficient. The obtained solutions turn out to be Malliavin differentiable and are used to derive a Bismut–Elworthy–Li formula for solutions of the Kolmogorov equation. We emphasise that our approach exhibits high flexibility to study a variety of other types of stochastic (partial) differential equations as e.g. stochastic differential equations driven by fractional Brownian motion.

Résumé. Dans cet article, nous cherchons à utiliser un critère de compacité de Da Prato, Malliavin, Nualart pour les fonctionnelles browniennes de carré intégrable pour construire des solutions fortes d'EDS sous une condition d'intégrabilité sur le coefficient de dérive. Les solutions obtenues se révèlent être Malliavin-différentiables et sont utilisées pour dériver une formule Bismut–Elworthy–Li pour des solutions de l'équation de Kolmogorov. Nous soulignons que notre approche présente une grande souplesse pour étudier une variété d'autres types d'équations différentielles stochastiques (aux dérivées partielles) comme par exemple des équations différentielles stochastiques conduites par un mouvement brownien fractionnaire.

MSC: 60H10; 60H07; 60H40; 60J60

Keywords: Strong solutions of SDEs; Malliavin calculus; Kolmogorov equation; Bismut–Elworthy–Li formula; Singular drift coefficient

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Thick points of high-dimensional Gaussian free fields

Linan Chen[1]

Department of Mathematics and Statistics, McGill University, 805 Sherbrooke Street West, Montréal, QC, H3A 0B9, Canada.
E-mail: lnchen@math.mcgill.ca

Abstract. This work aims to extend the existing results on thick points of logarithmic-correlated Gaussian Free Fields to Gaussian random fields that are more singular. To be specific, we adopt a sphere averaging regularization to study polynomial-correlated Gaussian Free Fields in higher-than-two dimensions. Under this setting, we introduce the definition of thick points which, heuristically speaking, are points where the value of the Gaussian Free Field is unusually large. We then establish a result on the Hausdorff dimension of the sets containing thick points.

Résumé. Cet article a pour but d'étendre certains résultats existants sur les points épais de champs libres gaussiens a corrélation logarithmique, à des champs aléatoires gaussiens qui sont plus singuliers. Plus précisément, nous utilisons une moyenne sphérique pour étudier les champs libres gaussiens a corrélation polynomiale en dimension supérieure à 2. Dans ce contexte nous introduisons une définition des points épais qui, de manière heuristique, sont les points pour lesquels la valeur du champ libre gaussien est inhabituellement grande. Nous établissons un résultat sur la dimension de Hausdorff des ensembles contenant ces points épais.

MSC: 60G60; 60G15

Keywords: Gaussian free field; Polynomial singularity; Thick point; Hausdorff dimension

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The size of the last merger and time reversal in Λ-coalescents

Götz Kersting[a,1], Jason Schweinsberg[b,2] and Anton Wakolbinger[a,1]

[a]*Institut für Mathematik, Goethe Universität, Frankfurt am Main, Germany.*
E-mail: kersting@math.uni-frankfurt.de; wakolbin@math.uni-frankfurt.de
[b]*Department of Mathematics, University of California, San Diego, 9500 Gilman Drive, La Jolla, CA 92093-0112, USA.*
E-mail: jschwein@math.ucsd.edu

Abstract. We consider the number of blocks involved in the last merger of a Λ-coalescent started with n blocks. We give conditions under which, as $n \to \infty$, the sequence of these random variables (a) is tight, (b) converges in distribution to a finite random variable or (c) converges to infinity in probability. Our conditions are optimal for Λ-coalescents that have a dust component. For general Λ, we relate the three cases to the existence, uniqueness and non-existence of invariant measures for the dynamics of the block-counting process, and in case (b) investigate the time-reversal of the block-counting process back from the time of the last merger.

Résumé. Nous considérons le nombre de blocs impliqués dans le dernier regroupement d'un Λ-coalescent issu de n blocs. Nous donnons des conditions sous lesquelles, quand n tend vers l'infini, la suite de variables aléatoires (a) est tendue (b) converge en loi vers une variable aléatoire finie ou (c) converge vers l'infini en probabilité. Nos conditions sont optimales pour les Λ-coalescents qui ont une composante de poussière. Pour un Λ général, nous associons ces trois cas à l'existence, l'unicité et la non-existence d'une mesure invariante pour la dynamique du processus de comptage des blocs. Dans le cas (b), nous étudions le retourné en temps du processus de comptage des blocs depuis de le temps de dernier regroupement.

MSC: Primary 60J27; secondary 60K05; 60G51

Keywords: Λ-coalescent; Block-counting process; Renewal theory; Subordinator

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Optimal discretization of stochastic integrals driven by general Brownian semimartingale[1]

Emmanuel Gobet and Uladzislau Stazhynski

CMAP, Ecole Polytechnique and CNRS, Université Paris Saclay, Route de Saclay, 91128 Palaiseau cedex, France.
E-mail: emmanuel.gobet@polytechnique.edu; uladzislau.stazhynski@polytechnique.edu

Abstract. We study the optimal discretization error of stochastic integrals, driven by a multidimensional continuous Brownian semimartingale. In this setting we establish a pathwise lower bound for the renormalized quadratic variation of the error and we provide a sequence of discretization stopping times, which is asymptotically optimal. The latter is defined as hitting times of random ellipsoids by the semimartingale at hand. In comparison with previous available results, we allow a quite large class of semimartingales (relaxing in particular the non degeneracy conditions usually requested) and we prove that the asymptotic lower bound is attainable.

Résumé. Nous étudions l'erreur de discrétisation optimale d'intégrale stochastique, dirigée par une semimartingale brownienne continue multidimensionnelle. Dans ce cadre, nous déterminons une borne inférieure trajectorielle pour la variation quadratique de l'erreur renormalisée et nous fournissons une suite de temps d'arrêt de discrétisation, suite qui est asymptotiquement optimale. Cette dernière est définie explicitement à partir des temps d'atteinte d'ellipsoïdes aléatoires par la semimartingale sous-jacente. En comparaison avec les précédents résultats, nous considérons une très grande classe de semimartingales (relâchant en particulier les conditions de non dégénérescence qui étaient habituellement requises) et nous prouvons que la borne inférieure asymptotique est atteignable.

MSC: 60G40; 60F15; 60H05

Keywords: Discretization of stochastic integrals; Hitting times; Random ellipsoids; Almost sure convergence

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Low-rank diffusion matrix estimation for high-dimensional time-changed Lévy processes

Denis Belomestny[a,b] and Mathias Trabs[c]

[a] *Faculty of Mathematics, Duisburg-Essen University, Thea-Leymann-Str. 9, D-45127 Essen, Germany. E-mail: denis.belomestny@uni-due.de*
[b] *National Research University Higher School of Economics, Shabolovka, 26, 119049 Moscow, Russia*
[c] *Faculty of Mathematics, Universität Hamburg, Bundesstraße 55, 20146 Hamburg, Germany. E-mail: mathias.trabs@uni-hamburg.de*

Abstract. The estimation of the diffusion matrix Σ of a high-dimensional, possibly time-changed Lévy process is studied, based on discrete observations of the process with a fixed distance. A low-rank condition is imposed on Σ. Applying a spectral approach, we construct a weighted least-squares estimator with nuclear-norm-penalisation. We prove oracle inequalities and derive convergence rates for the diffusion matrix estimator. The convergence rates show a surprising dependency on the rank of Σ and are optimal in the minimax sense for fixed dimensions. Theoretical results are illustrated by a simulation study.

Résumé. Nous étudions le problème de l'estimation de la matrice de diffusion Σ d'un processus de Lévy en grande dimension, qui peut être changé de temps, en se basant sur des observations discrètes du processus à une distance fixée. Nous imposons une condition de faible rang sur Σ. À l'aide d'une méthode spectrale, nous construisons un estimateur pondéré des moindres carrés avec une pénalisation par une norme nucléaire. Nous prouvons des inégalités oracle et obtenons des vitesses de convergence pour l'estimateur de la matrice de diffusion. Nous constatons que ces vitesses dépendent du rang de Σ d'une façon surprenante, et qu'elles sont optimales au sens minimax pour une dimension fixée. Ces résultats théoriques sont illustrés par une étude de simulations.

MSC: Primary 62M05; secondary 60G51; 62G05; 62M15

Keywords: Volatility estimation; Lasso-type estimator; Minimax convergence rates; Nonlinear inverse problem; Oracle inequalities; Time-changed Lévy process

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The near-critical Gibbs measure of the branching random walk

Michel Pain

Université Pierre et Marie Curie & École Normale Supérieure, Paris. E-mail: michel.pain@ens.fr

Abstract. Consider the supercritical branching random walk on the real line in the boundary case and the associated Gibbs measure $\nu_{n,\beta}$ on the nth generation, which is also the polymer measure on a disordered tree with inverse temperature β. The convergence of the partition function $W_{n,\beta}$, after rescaling, towards a nontrivial limit has been proved by Aïdékon and Shi (*Ann. Probab.* **42** (3) (2014) 959–993) in the critical case $\beta = 1$ and by Madaule (*J. Theoret. Probab.* **30** (1) (2017) 27–63) when $\beta > 1$. We study here the near-critical case, where $\beta_n \to 1$, and prove the convergence of W_{n,β_n}, after rescaling, towards a constant multiple of the limit of the derivative martingale. Moreover, trajectories of particles chosen according to the Gibbs measure $\nu_{n,\beta}$ have been studied by Madaule (*Stochastic Process. Appl.* **126** (2) (2016) 470–502) in the critical case, with convergence towards the Brownian meander, and by Chen, Madaule and Mallein (On the trajectory of an individual chosen according to supercritical gibbs measure in the branching random walk (2015) Preprint) in the strong disorder regime, with convergence towards the normalized Brownian excursion. We prove here the convergence for trajectories of particles chosen according to the near-critical Gibbs measure and display continuous families of processes from the meander to the excursion or to the Brownian motion.

Résumé. Considérons une marche aléatoire branchante surcritique réelle dans le cas frontière et la mesure de Gibbs associée $\nu_{n,\beta}$ sur la n-ième génération, qui est aussi la mesure de polymère sur un arbre désordonné avec température inverse β. La convergence de la fonction de partition $W_{n,\beta}$, après renormalisation, vers une limite non-triviale a été démontrée par Aïdékon et Shi (*Ann. Probab.* **42** (3) (2014) 959–993) dans le cas critique $\beta = 1$ et par Madaule (*J. Theoret. Probab.* **30** (1) (2017) 27–63) pour $\beta > 1$. On s'intéresse ici au cas presque-critique, où $\beta_n \to 1$, et on montre la convergence de W_{n,β_n}, après renormalisation, vers la limite de la martingale dérivée à un facteur multiplicatif près. D'autre part, les trajectoires de particules tirées selon la mesure de Gibbs $\nu_{n,\beta}$ ont été étudiées par Madaule (*Stochastic Process. Appl.* **126** (2) (2016) 470–502) dans le cas critique, avec convergence vers le méandre brownien, et par Chen, Madaule et Mallein (On the trajectory of an individual chosen according to supercritical gibbs measure in the branching random walk (2015) Preprint) dans le régime de désordre fort, avec convergence vers l'excursion brownienne. On montre ici la convergence des trajectoires de particules tirées selon la mesure de Gibbs presque-critique et cela fait apparaître une famille continue de processus allant du méandre jusqu'à l'excursion ou jusqu'au mouvement brownien.

MSC: 60J80; 60F05; 60F17

Keywords: Branching random walk; Additive martingale; Trajectories; Phase transition

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Characterization of a class of weak transport-entropy inequalities on the line[1]

Nathael Gozlan[a], Cyril Roberto[b], Paul-Marie Samson[a], Yan Shu[b] and Prasad Tetali[c,2]

[a]*Laboratoire d'Analyse et de Mathématiques Appliquées (UMR CNRS 8050), Université Paris Est Marne la Vallée, 5 bd Descartes, 77454 Marne la Vallée Cedex 2, France. E-mail: natael.gozlan@u-pem.fr; paul-marie.samson@u-pem.fr*
[b]*Laboratoire Modal'X, EA 3454, Université Paris Ouest Nanterre La Défense, 200 avenue de la République, 92000 Nanterre, France. E-mail: c.roberto@math.cnrs.fr; yan.shu@u-paris10.fr*
[c]*School of Mathematics & School of Computer Science, Georgia Institute of Technology, Atlanta, GA 30332, USA. E-mail: tetali@math.gatech.edu*

Abstract. We study an weak transport cost related to the notion of convex order between probability measures. On the real line, we show that this weak transport cost is reached for a coupling that does not depend on the underlying cost function. As an application, we give a necessary and sufficient condition for weak transport-entropy inequalities (related to concentration of convex/concave functions) to hold on the line. In particular, we obtain a weak transport-entropy form of the convex Poincaré inequality in dimension one.

Résumé. Dans cet article, nous étudions une nouvelle famille de coûts de transport optimaux faibles en lien avec la notion d'ordre convexe pour les mesures de probabilité. Nous montrons, en dimension un, que le couplage optimal ne dépend pas de la fonction de coût choisie. Nous utilisons ensuite ce résultat pour établir une condition nécessaire et suffisante pour les inégalités de transport-entropie associées à ces coûts de transport faibles. En particulier, nous obtenons une forme transport équivalente de l'inégalité de Poincaré restreinte aux fonctions convexes sur la droite.

MSC: 60E15; 32F32; 26D10

Keywords: Transport inequalities; Concentration of measure; Majorization

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Liouville quantum gravity on the unit disk

Yichao Huang[a], Rémi Rhodes[b] and Vincent Vargas[c]

[a]*ENS Ulm, DMA, 45 rue d'Ulm, 75005 Paris, France*
[b]*Université Paris-Est Marne la Vallée, LAMA, Champs sur Marne, France*
[c]*ENS Ulm, DMA, 45 rue d'Ulm, 75005 Paris, France*

Abstract. Our purpose is to pursue the rigorous construction of Liouville Quantum Field Theory on Riemann surfaces initiated by F. David, A. Kupiainen and the last two authors in the context of the Riemann sphere and inspired by the 1981 seminal work by Polyakov. In this paper, we investigate the case of simply connected domains with boundary. We also make precise conjectures about the relationship of this theory to scaling limits of random planar maps with boundary conformally embedded onto the disk.

Résumé. Notre but est d'étendre la construction rigoureuse de la Théorie Quantique des Champs de Liouville sur les surfaces de Riemann, initiée par F. David, A. Kupiainen et les deux derniers auteurs dans le contexte de la sphère de Riemann et inspirée par le travail pionnier de Polyakov en 1981. Dans ce papier nous étudions la théorie dans le cas de domaines simplement connexes à bord. Nous formulons également des conjectures précises sur la relation entre cette théorie et les limites d'échelle des grandes cartes planaires aléatoires à bord conformément plongées dans le disque unité.

MSC: 60D05; 81T40; 81T20

Keywords: Liouville Quantum Gravity; Quantum field theory; Gaussian multiplicative chaos; KPZ formula; KPZ scaling laws; Polyakov formula; Conformal anomaly

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Interpolation process between standard diffusion and fractional diffusion[1]

Cédric Bernardin[a,2], Patrícia Gonçalves[b,3], Milton Jara[c,4] and Marielle Simon[d,e,5]

[a]*Université Côte d'Azur, CNRS, LJAD, Parc Valrose, 06108 NICE Cedex 02, France. E-mail: cbernard@unice.fr*
[b]*Center for Mathematical Analysis, Geometry and Dynamical Systems, Instituto Superior Técnico, Universidade de Lisboa, Av. Rovisco Pais, 1049-001 Lisboa, Portugal. E-mail: patricia.goncalves@math.tecnico.ulisboa.pt*
[c]*Instituto de Matemática Pura e Aplicada, Estrada Dona Castorina 110, 22460-320 Rio De Janeiro, Brazil. E-mail: mjara@impa.br*
[d]*Inria Lille – Nord Europe, 40 avenue du Halley, 59650 Villeneuve d'Ascq, France. E-mail: marielle.simon@inria.fr*
[e]*Laboratoire Paul Painlevé, UMR CNRS 8524, Cité Scientifique, 59655 Villeneuve d'Ascq, France*

Abstract. We consider a Hamiltonian lattice field model with two conserved quantities, energy and volume, perturbed by stochastic noise preserving the two previous quantities. It is known that this model displays anomalous diffusion of energy of fractional type due to the conservation of the volume (*Nonlinearity* **25** (4) (2012) 1099–1133; *Arch. Ration. Mech. Anal.* **220** (2) (2016) 505–542). We superpose to this system a second stochastic noise conserving energy but not volume. If the intensity of this noise is of order one, normal diffusion of energy is restored while it is without effect if intensity is sufficiently small. In this paper we investigate the nature of the energy fluctuations for a critical value of the intensity. We show that the latter are described by an Ornstein–Uhlenbeck process driven by a Lévy process which interpolates between Brownian motion and the maximally asymmetric $3/2$-stable Lévy process. This result extends and solves a problem left open in (*J. Stat. Phys.* **159** (6) (2015) 1327–1368).

Résumé. Nous considérons un modèle de champs sur réseau Hamiltonien avec deux quantités conservées, l'énergie et le volume, perturbé par un bruit stochastique conservant les deux quantités précédentes. Il est connu que ce modèle produit une diffusion anormale de l'énergie de type fractionnaire en raison de la conservation du volume (*Nonlinearity* **25** (4) (2012) 1099–1133; *Arch. Ration. Mech. Anal.* **220** (2) (2016) 505–542). Nous superposons à cette dynamique un second bruit stochastique conservant l'énergie mais pas le volume. Si l'intensité de ce bruit est d'ordre 1, la diffusion normale de l'énergie est restaurée tandis qu'elle est sans effet si l'intensité est suffisamment faible. Dans ce papier nous étudions la nature des fluctuations d'énergie pour une valeur critique de l'intensité. Nous montrons que ces dernières sont décrites par un processus d'Ornstein–Uhlenbeck dirigé par un processus de Lévy qui interpole entre le mouvement Brownien et le processus de Lévy stable 3/2 totalement asymétrique. Ce résultat étend et résout un problème laissé ouvert dans (*J. Stat. Phys.* **159** (6) (2015) 1327–1368).

MSC: 60K35; 82C22; 82C44; 60G22; 74A25

Keywords: Anomalous diffusion; Chain of oscillators; Equilibrium fluctuations; Lévy process

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