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MEAN FIELD GAME OF MUTUAL HOLDING

BY MAO FABRICE DJETE[a] AND NIZAR TOUZI[b]

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We introduce a mean field model for optimal holding of a representative agent of her peers as a natural expected scaling limit from the corresponding N-agent model. The induced mean field dynamics appear naturally in a form which is not covered by standard McKean–Vlasov stochastic differential equations. We study the corresponding mean field game of mutual holding in the absence of common noise. Our first main result provides an explicit equilibrium of this mean field game, defined by a bang-bang control consisting in holding those competitors with positive drift coefficient of their dynamic value. We next use this mean field game equilibrium to construct (approximate) Nash equilibria for the corresponding N-player game. We also provide some numerical illustrations of our mean field game equilibrium which highlight some unexpected effects induced by our results.

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DISCRETE STICKY COUPLINGS OF FUNCTIONAL AUTOREGRESSIVE PROCESSES

BY ALAIN DURMUS[1,a], ANDREAS EBERLE[2,b], AURÉLIEN ENFROY[3,c], ARNAUD GUILLIN[4,d] AND PIERRE MONMARCHÉ[5,e]

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In this paper, we provide bounds in Wasserstein and total variation distances between the distributions of the successive iterates of two functional autoregressive processes with isotropic Gaussian noise of the form $Y_{k+1} = \mathrm{T}_\gamma(Y_k) + \sqrt{\gamma\sigma^2} Z_{k+1}$ and $\tilde{Y}_{k+1} = \tilde{\mathrm{T}}_\gamma(\tilde{Y}_k) + \sqrt{\gamma\sigma^2}\tilde{Z}_{k+1}$. More precisely, we give nonasymptotic bounds on $\rho(\mathcal{L}(Y_k), \mathcal{L}(\tilde{Y}_k))$, where ρ is an appropriate weighted Wasserstein distance or a V-distance, uniformly in the parameter γ, and on $\rho(\pi_\gamma, \tilde{\pi}_\gamma)$, where π_γ and $\tilde{\pi}_\gamma$ are the respective stationary measures of the two processes. The class of considered processes encompasses the Euler–Maruyama discretization of Langevin diffusions and its variants. The bounds we derive are of order γ as $\gamma \to 0$. To obtain our results, we rely on the construction of a discrete sticky Markov chain $(W_k^{(\gamma)})_{k\in\mathbb{N}}$ which bounds the distance between an appropriate coupling of the two processes. We then establish stability and quantitative convergence results for this process uniformly on γ. In addition, we show that it converges in distribution to the continuous sticky process studied in Howitt (Ph.D. thesis (2007)) and Eberle and Zimmer (*Ann. Inst. Henri Poincaré Probab. Stat.* **55** (2019) 2370–2394). Finally, we apply our result to Bayesian inference of ODE parameters and numerically illustrate them on two particular problems.

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ON r-TO-p NORMS OF RANDOM MATRICES WITH NONNEGATIVE ENTRIES: ASYMPTOTIC NORMALITY AND ℓ_∞-BOUNDS FOR THE MAXIMIZER

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For an $n \times n$ matrix A_n, the $r \to p$ operator norm is defined as

$$\|A_n\|_{r\to p} := \sup_{\boldsymbol{x}\in\mathbb{R}^n:\|\boldsymbol{x}\|_r\le 1} \|A_n\boldsymbol{x}\|_p \quad \text{for } r, p \ge 1.$$

For different choices of r and p, this norm corresponds to key quantities that arise in diverse applications including matrix condition number estimation, clustering of data, and construction of oblivious routing schemes in transportation networks. This article considers $r \to p$ norms of symmetric random matrices with nonnegative entries, including adjacency matrices of Erdős–Rényi random graphs, matrices with positive sub-Gaussian entries, and certain sparse matrices. For $1 < p \le r < \infty$, the asymptotic normality, as $n \to \infty$, of the appropriately centered and scaled norm $\|A_n\|_{r\to p}$ is established. When $p \ge 2$, this is shown to imply, as a corollary, asymptotic normality of the solution to the ℓ_p quadratic maximization problem, also known as the ℓ_p Grothendieck problem. Furthermore, a sharp ℓ_∞-approximation bound for the unique maximizing vector in the definition of $\|A_n\|_{r\to p}$ is obtained, and may be viewed as an ℓ_∞-stability result of the maximizer under random perturbations of the matrix with mean entries. This result, which may be of independent interest, is in fact shown to hold for a broad class of deterministic sequences of matrices having certain asymptotic expansion properties. The results obtained can be viewed as a generalization of the seminal results of Füredi and Komlós (1981) on asymptotic normality of the largest singular value of a class of symmetric random matrices, which corresponds to the special case $r = p = 2$ considered here. In the general case with $1 < p \le r < \infty$, spectral methods are no longer applicable, and so a new approach is developed involving a refined convergence analysis of a nonlinear power method and a perturbation bound on the maximizing vector, which may be of independent interest.

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SOLVING NON-MARKOVIAN STOCHASTIC CONTROL PROBLEMS DRIVEN BY WIENER FUNCTIONALS

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In this article, we present a general methodology for stochastic control problems driven by the Brownian motion filtration including non-Markovian and nonsemimartingale state processes controlled by mutually singular measures. The main result of this paper is the development of a numerical scheme for computing near-optimal controls associated with controlled Wiener functionals via a finite-dimensional approximation procedure. The approach does not require functional differentiability assumptions on the value process and ellipticity conditions on the diffusion components. The general convergence of the method is established under rather weak conditions for distinct types of non-Markovian and nonsemimartingale states. Explicit rates of convergence are provided in case the control acts only on the drift component of the controlled system. Near-closed/open-loop optimal controls are fully characterized by a dynamic programming algorithm and they are classified according to the strength of the possibly underlying non-Markovian memory. The theory is applied to stochastic control problems based on path-dependent SDEs and rough stochastic volatility models, where both drift and possibly degenerated diffusion components are controlled. Optimal control of drifts for nonlinear path-dependent SDEs driven by fractional Brownian motion with exponent $H \in (0, \frac{1}{2})$ is also discussed. Finally, we present a simple numerical example to illustrate the method.

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AZADKIA–CHATTERJEE'S CORRELATION COEFFICIENT ADAPTS TO MANIFOLD DATA

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In their seminal work, Azadkia and Chatterjee (*Ann. Statist.* **49** (2021) 3070–3102) initiated graph-based methods for measuring variable dependence strength. By appealing to nearest neighbor graphs based on the Euclidean metric, they gave an elegant solution to a problem of Rényi (*Acta Math. Acad. Sci. Hung.* **10** (1959) 441–451). This idea was later developed in Deb, Ghosal and Sen (2020) (https://arxiv.org/abs/2010.01768) and the authors there proved that, quite interestingly, Azadkia and Chatterjee's correlation coefficient can automatically adapt to the manifold structure of the data. This paper furthers their study in terms of calculating the statistic's limiting variance under independence—showing that it only depends on the manifold dimension—and extending this distribution-free property to a class of metrics beyond the Euclidean.

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SEPARATION CUTOFF FOR ACTIVATED RANDOM WALKS

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We consider activated random walks on arbitrary finite networks, with particles being inserted at random and absorbed at the boundary. Despite the nonreversibility of the dynamics and the lack of knowledge on the stationary distribution, we explicitly determine the relaxation time of the process, and prove that separation cutoff is equivalent to the product condition. We also provide sharp estimates on the center and width of the cutoff window. Finally, we illustrate those results by establishing explicit separation cutoffs on various networks, including: (i) large finite subgraphs of any fixed infinite nonamenable graph, with absorption at the boundary and (ii) large finite vertex-transitive graphs with absorption at a single vertex. The latter result settles a conjecture of Levine and Liang. Our proofs rely on the refined analysis of a strong stationary time recently discovered by Levine and Liang and involving the IDLA process.

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MEAN-FIELD ANALYSIS FOR LOAD BALANCING ON SPATIAL GRAPHS

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The analysis of large-scale, parallel-server load balancing systems has relied heavily on mean-field analysis. A pivotal assumption for this framework is that servers are exchangeable. However, modern data-centers have *data locality constraints*, such that tasks of a particular type can only be routed to a small subset of servers. An emerging line of research, therefore, considers load balancing algorithms on bipartite graphs where vertices represent task types and servers, respectively. Due to the lack of exchangeability in this model, mean-field techniques fundamentally break down. Recent progress has been made on graphs with strong edge-expansion properties, that is, where *any* two large subsets of vertices are well-connected. However, data locality often leads to spatial graphs that do not have strong expansion properties.

In this paper, we develop a novel coupling-based approach to establish mean-field approximation for a large class of graphs which includes spatial graphs. The method extends the scope of mean-field analysis far beyond the classical full-flexibility setup. En route, we prove that, starting from suitable states, the occupancy process becomes close to its steady state in a time that is independent of system size, which might be of independent interest. Numerical experiments are conducted, which positively support the theoretical results.

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ON LOEWNER CHAINS DRIVEN BY SEMIMARTINGALES AND COMPLEX BESSEL-TYPE SDES

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We prove existence (and simpleness) of the trace for both forward and backward Loewner chains under fairly general conditions on semimartingale drivers. As an application, we show that stochastic Komatu–Loewner evolutions $\mathrm{SKLE}_{\alpha,b}$ are generated by curves. As another application, motivated by a question of A. Sepúlveda, we show that for $\alpha > 3/2$ and Brownian motion B, the driving function $|B_t|^\alpha$ generates a simple curve for small t. On a related note we also introduce a complex variant of Bessel-type SDEs and prove existence and uniqueness of strong solution. Such SDEs appear naturally while describing the trace of Loewner chains. In particular, we write SLE_κ, $\kappa < 4$, in terms of stochastic flow of such SDEs.

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ON HIGH-DIMENSIONAL WAVELET EIGENANALYSIS

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In this paper, we characterize the asymptotic and large scale behavior of the eigenvalues of wavelet random matrices in high dimensions. We assume that possibly non-Gaussian, finite-variance p-variate measurements are made of a low-dimensional r-variate ($r \ll p$) fractional stochastic process with noncanonical scaling coordinates and in the presence of additive high-dimensional noise. The measurements are correlated both timewise and between rows. We show that the r largest eigenvalues of the wavelet random matrices, when appropriately rescaled, converge in probability to scale-invariant functions in the high-dimensional limit. By contrast, the remaining $p - r$ eigenvalues remain bounded in probability. Under additional assumptions, we show that the r largest log-eigenvalues of wavelet random matrices exhibit asymptotically Gaussian distributions. The results have direct consequences for statistical inference.

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DOMAIN OF ATTRACTION OF THE FIXED POINTS OF BRANCHING BROWNIAN MOTION

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We give a complete characterization of the domain of attraction of fixed points of branching Brownian motion (BBM) with critical drift. Prior to this classification, we introduce a suitable metric space of locally finite point measures on which we prove 1) that the BBM with critical drift is a well-defined Markov process and 2) that it satisfies the Feller property. Several applications of this characterization are given.

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THE COALESCENT STRUCTURE OF GALTON–WATSON TREES IN VARYING ENVIRONMENTS

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We investigate the genealogy of a sample of $k \geq 2$ particles chosen uniformly without replacement from a population alive at large times in a critical discrete-time Galton–Watson process in a varying environment (GWVE). We will show that subject to an explicit deterministic time-change involving only the mean and variances of the varying offspring distributions, the sample genealogy always converges to the same universal genealogical structure; it has the same tree topology as Kingman's coalescent, and the coalescent times of the $k-1$ pairwise mergers look like a mixture of independent identically distributed times. Our approach uses k distinguished *spine* particles and a suitable change of measure under which (a) the spines form a uniform sample without replacement, as required, but additionally (b) there is k-size biasing and discounting according to the population size. Our work significantly extends the spine techniques developed in Harris, Johnston and Roberts *(Ann. Appl. Probab.* (2020) **30** 1368–1414) for genealogies of uniform samples of size k in near-critical continuous-time Galton–Watson processes, as well as a two-spine GWVE construction in Cardona and Palau (*Bernoulli* (2021) **27** 1643–1665). Our results complement recent works by Kersting (*Proc. Steklov Inst. Maths.* (2022) **316** 209–219) and Boenkost, Foutel-Rodier and Schertzer (arXiv:2207.11612).

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GIBBS EQUILIBRIUM FLUCTUATIONS OF POINT VORTEX DYNAMICS

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We consider a system of N point vortices in a bounded domain with null total circulation, whose statistics are given by the canonical Gibbs ensemble at inverse temperature $\beta \geq 0$. We prove that the space-time fluctuation field around the (constant) mean field limit satisfies when $N \to \infty$ a generalized version of two-dimensional Euler dynamics preserving the Gaussian energy-enstrophy ensemble.

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LIMIT THEOREMS FOR ADDITIVE FUNCTIONALS OF SOME SELF-SIMILAR GAUSSIAN PROCESSES

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Under certain mild conditions, limit theorems for additive functionals of some d-dimensional self-similar Gaussian processes are obtained. These limit theorems work for general Gaussian processes including fractional Brownian motions, subfractional Brownian motions and bi-fractional Brownian motions. To prove these results, we use the method of moments and an enhanced chaining argument. The Gaussian processes under consideration are required to satisfy certain strong local nondeterminism property. A tractable sufficient condition for the strong local nondeterminism property is given and it only relays on the covariance functions of the Gaussian processes. Moreover, we give a sufficient condition for the distribution function of a random vector to be determined by its moments.

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INCREASING PATHS IN RANDOM TEMPORAL GRAPHS

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We consider random temporal graphs, a version of the classical Erdős–Rényi random graph $G(n, p)$ where additionally, each edge has a distinct random time stamp, and connectivity is constrained to sequences of edges with increasing time stamps. We study the asymptotics for the distances in such graphs, mostly in the regime of interest where np is of order $\log n$. We establish the first order asymptotics for the lengths of increasing paths: the lengths of the shortest and longest paths between typical vertices, the maxima of these lengths from a given vertex, as well as the maxima between any two vertices; this covers the (temporal) diameter.

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CONTINUOUS-TIME WEAKLY SELF-AVOIDING WALK ON $\mathbb{Z}$ HAS STRICTLY MONOTONE ESCAPE SPEED

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Weakly self-avoiding walk (WSAW) is a model of simple random walk paths that penalizes self-intersections. On $\mathbb{Z}$, Greven and den Hollander proved in 1993 that the discrete-time weakly self-avoiding walk has an asymptotically deterministic escape speed, and they conjectured that this speed should be strictly increasing in the repelling strength parameter. We study a continuous-time version of the model, give a different existence proof for the speed, and prove the speed to be strictly increasing. The proof uses a transfer matrix method implemented via a supersymmetric version of the BFS–Dynkin isomorphism theorem, spectral theory, Tauberian theory, and stochastic dominance.

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MARTINGALE TRANSPORTS AND MONGE MAPS

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It is well known that martingale transport plans between marginals $\mu \neq \nu$ are never given by Monge maps—with the understanding that the map is over the first marginal μ, or forward in time. Here, we change the perspective, with surprising results. We show that any distributions μ, ν in convex order with ν atomless admit a martingale coupling given by a Monge map over the *second* marginal ν. Namely, we construct a particular coupling called the barcode transport. Much more generally, we prove that such "backward Monge" martingale transports are dense in the set of all martingale couplings, paralleling the classical denseness result for Monge transports in the Kantorovich formulation of optimal transport. Various properties and applications are presented, including a refined version of Strassen's theorem and a mimicking theorem where the marginals of a given martingale are reproduced by a "backward deterministic" martingale, a remarkable type of process whose current state encodes its whole history.

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FULL Γ-EXPANSION OF REVERSIBLE MARKOV CHAINS LEVEL TWO LARGE DEVIATIONS RATE FUNCTIONALS

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Let $\Xi_n \subset \mathbb{R}^d$, $n \ge 1$, be a sequence of finite sets and consider a Ξ_n-valued, irreducible, reversible, continuous-time Markov chain $(X_t^{(n)} : t \ge 0)$. Denote by $\mathscr{P}(\mathbb{R}^d)$ the set of probability measures on $\mathbb{R}^d$ and by $\mathscr{I}_n : \mathscr{P}(\mathbb{R}^d) \to [0, +\infty)$ the level two large deviations rate functional for $X_t^{(n)}$ as $t \to \infty$. We present a general method, based on tools used to prove the metastable behaviour of Markov chains, to derive a full expansion of $\mathscr{I}_n$ expressing it as $\mathscr{I}_n = \mathscr{I}^{(0)} + \sum_{1 \le p \le \mathfrak{q}} (1/\theta_n^{(p)}) \mathscr{I}^{(p)}$, where $\mathscr{I}^{(p)} : \mathscr{P}(\mathbb{R}^d) \to [0, +\infty]$ represent rate functionals independent of n and $\theta_n^{(p)}$ sequences such that $\theta_n^{(1)} \to \infty$, $\theta_n^{(p)}/\theta_n^{(p+1)} \to 0$ for $1 \le p < \mathfrak{q}$. The speed $\theta_n^{(p)}$ corresponds to the time-scale at which the Markov chains $X_t^{(n)}$ exhibits a metastable behaviour, and the $\mathscr{I}^{(p-1)}$ zero-level sets to the metastable states. To illustrate the theory we apply the method to random walks in potential fields.

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POPULATION DYNAMICS UNDER DEMOGRAPHIC AND ENVIRONMENTAL STOCHASTICITY

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The present paper is devoted to the study of the long term dynamics of diffusion processes modelling a single species that experiences both demographic and environmental stochasticity. In our setting, the long term dynamics of the diffusion process in the absence of demographic stochasticity is determined by the sign of Λ_0, the external Lyapunov exponent, as follows: $\Lambda_0 < 0$ implies (asymptotic) extinction and $\Lambda_0 > 0$ implies convergence to a unique positive stationary distribution μ_0. If the system is of size $\frac{1}{\epsilon^2}$ for small $\epsilon > 0$ (the intensity of demographic stochasticity), demographic effects will make the extinction time finite almost surely. This suggests that to understand the dynamics one should analyze the quasi-stationary distribution (QSD) μ_ϵ of the system. The existence and uniqueness of the QSD is well known under mild assumptions.

We look at what happens when the population size is sent to infinity, that is, when $\epsilon \to 0$. We show that the external Lyapunov exponent still plays a key role: (1) If $\Lambda_0 < 0$, then $\mu_\epsilon \to \delta_0$, the mean extinction time is of order $|\ln \epsilon|$ and the extinction rate associated with the QSD μ_ϵ has a lower bound of order $\frac{1}{|\ln \epsilon|}$; (2) If $\Lambda_0 > 0$, then $\mu_\epsilon \to \mu_0$, the mean extinction time is polynomial in $\frac{1}{\epsilon^2}$ and the extinction rate is polynomial in ϵ^2. Furthermore, when $\Lambda_0 > 0$ we are able to show that the system exhibits multiscale dynamics: at first the process quickly approaches the QSD μ_ϵ and then, after spending a polynomially long time there, it relaxes to the extinction state. We give sharp asymptotics in ϵ for the time spent close to μ_ϵ.

In contrast to models that only take into account demographic stochasticity, our results demonstrate the significant effect of environmental stochasticity—it turns an exponentially long mean extinction time to a subexponential one.

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THE EXPECTED EULER CHARACTERISTIC APPROXIMATION TO EXCURSION PROBABILITIES OF GAUSSIAN VECTOR FIELDS

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Let $\{(X(t), Y(s)) : t \in T, s \in S\}$ be an $\mathbb{R}^2$-valued, centered, unit-variance smooth Gaussian vector field, where T and S are compact rectangles in the Euclidean space. It is shown that, as $u \to \infty$, the joint excursion probability $\mathbb{P}\{\sup_{t\in T} X(t) \ge u, \sup_{s\in S} Y(s) \ge u\}$ can be approximated by $\mathbb{E}\{\chi(A_u)\}$, the expected Euler characteristic of the excursion set $A_u = \{(t, s) \in T \times S : X(t) \ge u, Y(s) \ge u\}$, such that the error is super-exponentially small. This verifies the expected Euler characteristic heuristic (cf. Taylor, Takemura and Alder (2005), Alder and Taylor (2007)) for a large class of smooth Gaussian vector fields.

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STABILITY AND STATISTICAL INFERENCE FOR SEMIDISCRETE OPTIMAL TRANSPORT MAPS

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We study statistical inference for the optimal transport (OT) map (also known as the Brenier map) from a known absolutely continuous reference distribution onto an unknown finitely discrete target distribution. We derive limit distributions for the L^p-error with arbitrary $p \in [1, \infty)$ and for linear functionals of the empirical OT map, together with their moment convergence. The former has a non-Gaussian limit, whose explicit density is derived, while the latter attains asymptotic normality. For both cases, we also establish consistency of the nonparametric bootstrap. The derivation of our limit theorems relies on new stability estimates of functionals of the OT map with respect to the dual potential vector, which may be of independent interest. We also discuss applications of our limit theorems to the construction of confidence sets for the OT map and inference for a maximum tail correlation. Finally, we show that, while the empirical OT map does not possess nontrivial weak limits in the L^2 space, it satisfies a central limit theorem in a dual Hölder space, and the Gaussian limit law attains the asymptotic efficiency bound.

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SURVIVAL AND COMPLETE CONVERGENCE FOR A BRANCHING ANNIHILATING RANDOM WALK

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We study a discrete-time branching annihilating random walk (BARW) on the d-dimensional lattice. Each particle produces a Poissonian number of offspring with mean μ which independently move to a uniformly chosen site within a fixed distance R from their parent's position. Whenever a site is occupied by at least two particles, all the particles at that site are annihilated. We prove that for any $\mu > 1$ the process survives when R is sufficiently large. For fixed R we show that the process dies out if μ is too small or too large. Furthermore, we exhibit an interval of μ-values for which the process survives and possesses a unique nontrivial ergodic equilibrium for R sufficiently large. We also prove complete convergence for that case.

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LDP FOR INHOMOGENEOUS U-STATISTICS

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In this paper we derive a large deviation principle (LDP) for inhomogeneous U/V-statistics of a general order. Using this, we derive a LDP for two types of statistics: random multilinear forms, and number of monochromatic copies of a subgraph. We show that the corresponding rate functions in these cases can be expressed as a variational problem over a suitable space of functions. We use the tools developed to study Gibbs measures with the corresponding Hamiltonians, which include tensor generalizations of both Ising (with noncompact base measure) and Potts models. For these Gibbs measures, we establish scaling limits of log normalizing constants, and weak laws in terms of weak* topology, which are of possible independent interest.

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A TWO-TABLE THEOREM FOR A DISORDERED CHINESE RESTAURANT PROCESS

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We investigate a disordered variant of Pitman's Chinese restaurant process where tables carry i.i.d. weights. Incoming customers choose to sit at an occupied table with a probability proportional to the product of its occupancy and its weight, or they sit at an unoccupied table with a probability proportional to a parameter $\theta > 0$. This is a system out of equilibrium where the proportion of customers at any given table converges to zero almost surely. We show that for weight distributions in any of the three extreme value classes, Weibull, Gumbel or Fréchet, the proportion of customers sitting at the largest table converges to one in probability, but not almost surely, and the proportion of customers sitting at either of the largest two tables converges to one almost surely.

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SECOND ERRATA TO "OCCUPATION AND LOCAL TIMES FOR SKEW BROWNIAN MOTION WITH APPLICATIONS TO DISPERSION ACROSS AN INTERFACE"

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The failure of a symmetry argument for the Laplace transform of a multivariate density given in *Ann. Appld. Probab.* **21**(1) (2011), 183–214 was neglected in preparation of the first errata (*Ann. Appld. Probab.* **21**(5) (2011), 2050–2051). The formula for the Laplace transform is corrected in this errata.

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