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CLUSTER SIZE DISTRIBUTIONS OF EXTREME VALUES FOR THE POISSON–VORONOI TESSELLATION

BY NICOLAS CHENAVIER AND CHRISTIAN Y. ROBERT

Université du Littoral Côte d'Opale and Université Lyon 1

We consider the Voronoi tessellation based on a homogeneous Poisson point process in an Euclidean space. For a geometric characteristic of the cells (e.g., the inradius, the circumradius, the volume), we investigate the point process of the nuclei of the cells with large values. Conditions are obtained for the convergence in distribution of this point process of exceedances to a homogeneous compound Poisson point process. We provide a characterization of the asymptotic cluster size distribution which is based on the Palm version of the point process of exceedances. This characterization allows us to compute efficiently the values of the extremal index and the cluster size probabilities by simulation for various geometric characteristics. The extension to the Poisson–Delaunay tessellation is also discussed.

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LIMITING BEHAVIOR OF 3-COLOR EXCITABLE MEDIA ON ARBITRARY GRAPHS

BY JANKO GRAVNER*,[1], HANBAEK LYU† AND DAVID SIVAKOFF†,[2]

University of California, Davis and Ohio State University*†

Fix a simple graph $G = (V, E)$ and choose a random initial 3-coloring of vertices drawn from a uniform product measure. The 3-color cycle cellular automaton is a process in which at each discrete time step in parallel, every vertex with color i advances to the successor color $(i + 1) \bmod 3$ if in contact with a neighbor with the successor color, and otherwise retains the same color. In the Greenberg–Hastings model, the same update rule applies only to color 0, while other two colors automatically advance. The limiting behavior of these processes has been studied mainly on the integer lattices. In this paper, we introduce a monotone comparison process defined on the universal covering space of the underlying graph, and characterize the limiting behavior of these processes on arbitrary connected graphs. In particular, we establish a phase transition on the Erdős–Rényi random graph. On infinite trees, we connect the rate of color change to the cloud speed of an associated tree-indexed walk. We give estimates of the cloud speed by generalizing known results to trees with leaves.

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WEIGHTED MULTILEVEL LANGEVIN SIMULATION OF INVARIANT MEASURES

BY GILLES PAGÈS AND FABIEN PANLOUP

UPMC and Université d'Angers

We investigate a weighted multilevel Richardson–Romberg extrapolation for the ergodic approximation of invariant distributions of diffusions adapted from the one introduced in [*Bernoulli* **23** (2017) 2643–2692] for regular Monte Carlo simulation. In a first result, we prove under weak confluence assumptions on the diffusion, that for any integer $R \geq 2$, the procedure allows us to attain a rate $n^{\frac{R}{2R+1}}$ whereas the original algorithm convergence is at a weak rate $n^{1/3}$. Furthermore, this is achieved without any explosion of the asymptotic variance. In a second part, under stronger confluence assumptions and with the help of some second-order expansions of the asymptotic error, we go deeper in the study by optimizing the choice of the parameters involved by the method. In particular, for a given $\varepsilon > 0$, we exhibit some semi-explicit parameters for which the number of iterations of the Euler scheme required to attain a mean-squared error lower than ε^2 is about $\varepsilon^{-2}\log(\varepsilon^{-1})$.

Finally, we numerically test this multilevel Langevin estimator on several examples including the simple one-dimensional Ornstein–Uhlenbeck process but also a high dimensional diffusion motivated by a statistical problem. These examples confirm the theoretical efficiency of the method.

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WRIGHT–FISHER DIFFUSIONS IN STOCHASTIC SPATIAL EVOLUTIONARY GAMES WITH DEATH–BIRTH UPDATING[1]

BY YU-TING CHEN

University of Tennessee

We investigate stochastic spatial evolutionary games with death–birth updating in large finite populations. Within growing spatial structures subject to appropriate conditions, the density processes of a fixed type are proven to converge to the one-dimensional Wright–Fisher diffusions. Convergence in the Wasserstein distance of the laws of the occupation measures also holds. The proofs study the convergences under certain voter models by an equivalence between their laws and the laws of the evolutionary games. In particular, the additional growing dimensions in minimal systems that close the dynamics of the game density processes are cut off in the limit.

As another application of this equivalence of laws, we consider a first-derivative test among the major methods for these evolutionary games in a large population of size N. Requiring only the assumption that the stationary probabilities of the corresponding voting kernel are comparable to uniform probabilities, we prove that the test is applicable at least up to weak selection strengths in the usual biological sense [i.e., selection strengths of the order $\mathcal{O}(1/N)$].

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IMPROVED BOUNDS FOR SPARSE RECOVERY FROM SUBSAMPLED RANDOM CONVOLUTIONS

BY SHAHAR MENDELSON[*,†,1], HOLGER RAUHUT[‡,1] AND RACHEL WARD[§,2]

Technion[*], *Australian National University*[†], *RWTH Aachen University*[‡] *and University of Texas at Austin*[§]

We study the recovery of sparse vectors from subsampled random convolutions via ℓ_1-minimization. We consider the setup in which both the subsampling locations as well as the generating vector are chosen at random. For a sub-Gaussian generator with independent entries, we improve previously known estimates: if the sparsity s is small enough, that is, $s \lesssim \sqrt{n/\log(n)}$, we show that $m \gtrsim s\log(en/s)$ measurements are sufficient to recover s-sparse vectors in dimension n with high probability, matching the well-known condition for recovery from standard Gaussian measurements. If s is larger, then essentially $m \geq s\log^2(s)\log(\log(s))\log(n)$ measurements are sufficient, again improving over previous estimates. Our results are shown via the so-called robust null space property which is weaker than the standard restricted isometry property. Our method of proof involves a novel combination of small ball estimates with chaining techniques which should be of independent interest.

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DIFFUSION LIMITED AGGREGATION ON THE BOOLEAN LATTICE

BY ALAN FRIEZE[1] AND WESLEY PEGDEN[2]

Carnegie Mellon University

In the Diffusion Limited Aggregation (DLA) process on $\mathbb{Z}^2$, or more generally $\mathbb{Z}^d$, particles aggregate to an initially occupied origin by arrivals on a random walk. The scaling limit of the result, empirically, is a fractal with dimension strictly less than d. Very little has been shown rigorously about the process, however.

We study an analogous process on the Boolean lattice $\{0, 1\}^n$, in which particles take random decreasing walks from $(1, \dots, 1)$, and stick at the last vertex before they encounter an occupied site for the first time; the vertex $(0, \dots, 0)$ is initially occupied. In this model, we can rigorously prove that lower levels of the lattice become full, and that the process ends by producing an isolated path of unbounded length reaching $(1, \dots, 1)$.

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VERIFICATION THEOREMS FOR STOCHASTIC OPTIMAL CONTROL PROBLEMS IN HILBERT SPACES BY MEANS OF A GENERALIZED DYNKIN FORMULA

BY SALVATORE FEDERICO AND FAUSTO GOZZI

Università di Siena and LUISS University

Verification theorems are key results to successfully employ the dynamic programming approach to optimal control problems. In this paper, we introduce a new method to prove verification theorems for infinite dimensional stochastic optimal control problems. The method applies in the case of additively controlled Ornstein–Uhlenbeck processes, when the associated Hamilton–Jacobi–Bellman (HJB) equation admits a *mild solution* (in the sense of [*J. Differential Equations* **262** (2017) 3343–3389]). The main methodological novelty of our result relies on the fact that it is not needed to prove, as in previous literature (see, e.g., [*Comm. Partial Differential Equations* **20** (1995) 775–826]), that the mild solution is a *strong solution*, that is, a suitable limit of classical solutions of approximating HJB equations. To achieve the goal, we prove a new type of Dynkin formula, which is the key tool for the proof of our main result.

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STABILITY CONDITIONS FOR A DISCRETE-TIME DECENTRALISED MEDIUM ACCESS ALGORITHM

BY SEVA SHNEER[1] AND ALEXANDER STOLYAR

Heriot-Watt University and University of Illinois at Urbana-Champaign

We consider a stochastic queueing system modelling the behaviour of a wireless network with nodes employing a discrete-time version of the standard decentralised medium access algorithm. The system is *unsaturated*—each node receives an exogenous flow of packets at the rate of λ packets per time slot. Each packet takes one slot to transmit, but neighbouring nodes cannot transmit simultaneously. The algorithm we study is *standard* in the following sense: a node with an empty queue does *not* compete for medium access; the access procedure by a node does *not* depend on its queue length as long as it is nonzero. Two system topologies are considered, with nodes arranged in a circle and in a line. We prove that, for either topology, the system is stochastically stable under the condition $\lambda < 2/5$. This result is intuitive for the circle topology as the throughput each node receives in the saturated system (with infinite queues) is equal to the so-called *parking constant*, which is larger than $2/5$. (This fact, however, does not help us to prove the result.) The result is not intuitive for the line topology as in the saturated system some nodes receive a throughput lower than $2/5$.

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CRAMÉR'S ESTIMATE FOR THE REFLECTED PROCESS REVISITED[1]

BY R. A. DONEY AND PHILIP S. GRIFFIN

University of Manchester and Syracuse University

The reflected process of a random walk or Lévy process arises in many areas of applied probability, and a question of particular interest is how the tail of the distribution of the heights of the excursions away from zero behaves asymptotically. The Lévy analogue of this is the tail behaviour of the characteristic measure of the height of an excursion. Apparently, the only case where this is known is when Cramér's condition hold. Here, we establish the asymptotic behaviour for a large class of Lévy processes, which have exponential moments but do not satisfy Cramér's condition. Our proof also applies in the Cramér case, and corrects a proof of this given in Doney and Maller [*Ann. Appl. Probab.* **15** (2005) 1445–1450].

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JUSTIFYING DIFFUSION APPROXIMATIONS FOR MULTICLASS QUEUEING NETWORKS UNDER A MOMENT CONDITION

BY HENG-QING YE[1] AND DAVID D. YAO[2]

Hong Kong Polytechnic University and Columbia University

Multiclass queueing networks (MQN) are, in general, difficult objects to study analytically. The diffusion approximation refers to using the stationary distribution of the diffusion limit as an approximation of the diffusion-scaled process (say, the workload) in the original MQN. To validate such an approximation amounts to justifying the interchange of two limits, $t \to \infty$ and $k \to \infty$, with t being the time index and k, the scaling parameter. Here, we show this interchange of limits is justified under a p^*th moment condition on the primitive data, the interarrival and service times; and we provide an explicit characterization of the required order (p^*), which depends naturally on the desired order of moment of the workload process.

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Key words and phrases. Multiclass queueing network, diffusion limit, interchange of limits, uniform stability.

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EXPONENTIAL RANDOM GRAPHS BEHAVE LIKE MIXTURES OF STOCHASTIC BLOCK MODELS

BY RONEN ELDAN[1,2] AND RENAN GROSS[1,3]

Weizmann Institute of Science

We study the behavior of exponential random graphs in both the sparse and the dense regime. We show that exponential random graphs are approximate mixtures of graphs with independent edges whose probability matrices are critical points of an associated functional, thereby satisfying a certain matrix equation. In the dense regime, every solution to this equation is close to a block matrix, concluding that the exponential random graph behaves roughly like a mixture of stochastic block models. We also show existence and uniqueness of solutions to this equation for several families of exponential random graphs, including the case where the subgraphs are counted with positive weights and the case where all weights are small in absolute value. In particular, this generalizes some of the results in a paper by Chatterjee and Diaconis from the dense regime to the sparse regime and strengthens their bounds from the cut-metric to the one-metric.

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TRACY–WIDOM FLUCTUATIONS FOR PERTURBATIONS OF THE LOG-GAMMA POLYMER IN INTERMEDIATE DISORDER

BY ARJUN KRISHNAN AND JEREMY QUASTEL

University of Rochester and University of Toronto

The free-energy fluctuations of the discrete directed polymer in $1+1$ dimensions is conjecturally in the Tracy–Widom universality class at all finite temperatures and in the intermediate disorder regime. Seppäläinen's log-gamma polymer was proven to have GUE Tracy–Widom fluctuations in a restricted temperature range by Borodin, Corwin and Remenik [*Comm. Math. Phys.* **324** (2013) 215–232]. We remove this restriction, and extend this result into the intermediate disorder regime. This result also identifies the scale of fluctuations of the log-gamma polymer in the intermediate disorder regime, and thus verifies a conjecture of Alberts, Khanin and Quastel [*Ann. Probab.* **42** (2014) 1212–1256]. Using a perturbation argument, we show that any polymer that matches a certain number of moments with the log-gamma polymer also has Tracy–Widom fluctuations in intermediate disorder.

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ON THE POLYNOMIAL CONVERGENCE RATE TO NONEQUILIBRIUM STEADY STATES

BY YAO LI

University of Massachusetts, Amherst

We consider a stochastic energy exchange model that models the 1-D microscopic heat conduction in the nonequilibrium setting. In this paper, we prove the existence and uniqueness of the nonequilibrium steady state (NESS) and, furthermore, the polynomial speed of convergence to the NESS. Our result shows that the asymptotic properties of this model and its deterministic dynamical system origin are consistent. The proof uses a new technique called the induced chain method. We partition the state space and work on both the Markov chain induced by an "active set" and the tail of return time to this "active set."

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PERFECT HEDGING IN ROUGH HESTON MODELS

BY OMAR EL EUCH AND MATHIEU ROSENBAUM

École Polytechnique

Rough volatility models are known to reproduce the behavior of historical volatility data while at the same time fitting the volatility surface remarkably well, with very few parameters. However, managing the risks of derivatives under rough volatility can be intricate since the dynamics involve fractional Brownian motion. We show in this paper that surprisingly enough, explicit hedging strategies can be obtained in the case of rough Heston models. The replicating portfolios contain the underlying asset and the forward variance curve, and lead to perfect hedging (at least theoretically). From a probabilistic point of view, our study enables us to disentangle the infinite-dimensional Markovian structure associated to rough volatility models.

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THE COLLISION SPECTRUM OF Λ-COALESCENTS[1]

BY ALEXANDER GNEDIN*, ALEXANDER IKSANOV†, ALEXANDER MARYNYCH†,2 AND MARTIN MÖHLE‡

Queen Mary University of London, Taras Shevchenko National University of Kyiv† and Eberhard Karls University of Tübingen‡*

Λ-coalescents model the evolution of a coalescing system in which any number of blocks randomly sampled from the whole may merge into a larger block. For the coalescent restricted to initially n singletons, we study the collision spectrum $(X_{n,k} : 2 \le k \le n)$, where $X_{n,k}$ counts, throughout the history of the process, the number of collisions involving exactly k blocks. Our focus is on the large n asymptotics of the joint distribution of the $X_{n,k}$'s, as well as on functional limits for the bulk of the spectrum for simple coalescents. Similar to the previous studies of the total number of collisions, the asymptotics of the collision spectrum largely depends on the behaviour of the measure Λ in the vicinity of 0. In particular, for beta(a, b)-coalescents different types of limit distributions occur depending on whether $0 < a \le 1$, $1 < a < 2$, $a = 2$ or $a > 2$.

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TAIL MEASURE AND SPECTRAL TAIL PROCESS OF REGULARLY VARYING TIME SERIES

BY CLÉMENT DOMBRY[1], ENKELEJD HASHORVA[2] AND PHILIPPE SOULIER[3]

Université Bourgogne Franche-Comté, Université de Lausanne and Université Paris Nanterre

The goal of this paper is an exhaustive investigation of the link between the tail measure of a regularly varying time series and its spectral tail process, independently introduced in [Owada and Samorodnitsky (2012)] and [*Stochastic Process. Appl.* **119** (2009) 1055–1080]. Our main result is to prove in an abstract framework that there is a one-to-one correspondence between these two objects, and given one of them to show that it is always possible to build a time series of which it will be the tail measure or the spectral tail process. For nonnegative time series, we recover results explicitly or implicitly known in the theory of max-stable processes.

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MUTATION FREQUENCIES IN A BIRTH–DEATH BRANCHING PROCESS

BY DAVID CHEEK[1] AND TIBOR ANTAL

University of Edinburgh

First, we revisit the stochastic Luria–Delbrück model: a classic two-type branching process which describes cell proliferation and mutation. We prove limit theorems and exact results for the mutation times, clone sizes and number of mutants. Second, we extend the framework to consider mutations at multiple sites along the genome. The number of mutants in the two-type model characterises the mean site frequency spectrum in the multiple-site model. Our predictions are consistent with previously published cancer genomic data.

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