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CONTENTS

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CONTENTS

(continued)

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Applying Dynkin's isomorphism: An alternative approach to understand the Markov property of the de Wijs process

DEBASHIS MONDAL

Department of Statistics, Oregon State University, 44 Kidder Hall, Corvallis, OR 97331, USA. E-mail: debashis@stat.oregonstate.edu

Dynkin's (*Bull. Amer. Math. Soc.* **3** (1980) 975–999) seminal work associates a multidimensional transient symmetric Markov process with a multidimensional Gaussian random field. This association, known as Dynkin's isomorphism, has profoundly influenced the studies of Markov properties of generalized Gaussian random fields. Extending Dykin's isomorphism, we study here a particular generalized Gaussian Markov random field, namely, the de Wijs process that originated in Georges Matheron's pioneering work on mining geostatistics and, following McCullagh (*Ann. Statist.* **30** (2002) 1225–1310), is now receiving renewed attention in spatial statistics. This extension of Dynkin's theory associates the de Wijs process with the (recurrent) Brownian motion on the two dimensional plane, grants us further insight into Matheron's kriging formula for the de Wijs process and highlights previously unexplored relationships of the central Markov models in spatial statistics with Markov processes on the plane.

Keywords: additive functions; Brownian motion; intrinsic autoregressions; kriging; potential kernel; random walk; screening effect; variogram

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1350-7265

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Adaptive MCMC with online relabeling

RÉMI BARDENET[1], OLIVIER CAPPÉ[2,*], GERSENDE FORT[2,**] and BALÁZS KÉGL[1,3]

[1]*Laboratoire de Recherche en Informatique, Université Paris-Sud XI, rue Noetzlin, 91190 Gif-sur-Yvette, France. E-mail: remi.bardenet@gmail.com*
[2]*LTCI, Telecom ParisTech & CNRS, 37 rue Dareau, 75013 Paris, France.*
E-mail: [*] *cappe@telecom-paristech.fr;* [**] *gfort@telecom-paristech.fr*
[3]*CNRS, Laboratoire de l'Accélérateur Linéaire, Université Paris-Sud XI, 91898 Orsay, France.*
E-mail: balazs.kegl@gmail.com

When targeting a distribution that is *artificially* invariant under some permutations, Markov chain Monte Carlo (MCMC) algorithms face the *label-switching* problem, rendering marginal inference particularly cumbersome. Such a situation arises, for example, in the Bayesian analysis of finite mixture models. Adaptive MCMC algorithms such as adaptive Metropolis (AM), which self-calibrates its proposal distribution using an online estimate of the covariance matrix of the target, are no exception. To address the label-switching issue, *relabeling* algorithms associate a permutation to each MCMC sample, trying to obtain reasonable marginals. In the case of adaptive Metropolis (*Bernoulli* **7** (2001) 223–242), an *online* relabeling strategy is required. This paper is devoted to the AMOR algorithm, a provably consistent variant of AM that can cope with the label-switching problem. The idea is to nest relabeling steps within the MCMC algorithm based on the estimation of a *single covariance matrix* that is used *both* for adapting the covariance of the proposal distribution in the Metropolis algorithm step *and* for online relabeling. We compare the behavior of AMOR to similar relabeling methods. In the case of compactly supported target distributions, we prove a strong law of large numbers for AMOR and its ergodicity. These are the first results on the consistency of an online relabeling algorithm to our knowledge. The proof underlines latent relations between relabeling and vector quantization.

Keywords: adaptive Markov chain Monte Carlo; label-switching; stochastic approximation; vector quantization

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Mimicking self-similar processes

JIE YEN FAN*, KAIS HAMZA** and FIMA KLEBANER†

School of Mathematical Sciences, Monash University, Clayton, VIC 3800, Australia.
*E-mail: *jieyen.fan@monash.edu; **kais.hamza@monash.edu; †fima.klebaner@monash.edu*

We construct a family of self-similar Markov martingales with given marginal distributions. This construction uses the self-similarity and Markov property of a reference process to produce a family of Markov processes that possess the same marginal distributions as the original process. The resulting processes are also self-similar with the same exponent as the original process. They can be chosen to be martingales under certain conditions. In this paper, we present two approaches to this construction, the transition-randomising approach and the time-change approach. We then compute the infinitesimal generators and obtain some path properties of the resulting processes. We also give some examples, including continuous Gaussian martingales as a generalization of Brownian motion, martingales of the squared Bessel process, stable Lévy processes as well as an example of an artificial process having the marginals of $t^{\kappa} V$ for some symmetric random variable V. At the end, we see how we can mimic certain Brownian martingales which are non-Markovian.

Keywords: Lévy processes; martingales with given marginals; self-similar

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Concentration inequalities for sampling without replacement

RÉMI BARDENET[1] and ODALRIC-AMBRYM MAILLARD[2]

[1]*Department of Statistics, University of Oxford, 1 South Parks Road, OX1 3TG Oxford, UK.*
E-mail: remi.bardenet@gmail.com
[2]*Faculty of Electrical Engineering, The Technion, Fishbach Building, 32000 Haifa, Israel.*
E-mail: odalric.maillard@ee.technion.ac.il

Concentration inequalities quantify the deviation of a random variable from a fixed value. In spite of numerous applications, such as opinion surveys or ecological counting procedures, few concentration results are known for the setting of sampling without replacement from a finite population. Until now, the best general concentration inequality has been a Hoeffding inequality due to Serfling [*Ann. Statist.* **2** (1974) 39–48]. In this paper, we first improve on the fundamental result of Serfling [*Ann. Statist.* **2** (1974) 39–48], and further extend it to obtain a Bernstein concentration bound for sampling without replacement. We then derive an empirical version of our bound that does not require the variance to be known to the user.

Keywords: Bernstein; concentration bounds; sampling without replacement; Serfling

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Lipschitz partition processes

HARRY CRANE

Department of Statistics & Biostatistics, Rutgers University, 110 Frelinghuysen Road, Hill Center, Room 461, Piscataway, NJ 08854, USA. E-mail: hcrane@stat.rutgers.edu

We introduce a family of Markov processes on set partitions with a bounded number of blocks, called *Lipschitz partition processes*. We construct these processes explicitly by a Poisson point process on the space of Lipschitz continuous maps on partitions. By this construction, the *Markovian consistency property* is readily satisfied; that is, the finite restrictions of any Lipschitz partition process comprise a compatible collection of finite state space Markov chains. We further characterize the class of exchangeable Lipschitz partition processes by a novel set-valued matrix operation.

Keywords: coalescent process; de Finetti's theorem; exchangeable random partition; iterated random functions; Markov process; paintbox process; Poisson random measure

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Qualitative robustness of statistical functionals under strong mixing

HENRYK ZÄHLE

Department of Mathematics, Saarland University, Postfach 151150, D-66041 Saarbrücken, Germany. E-mail: zaehle@math.uni-sb.de

A new concept of (asymptotic) qualitative robustness for plug-in estimators based on identically distributed possibly *dependent* observations is introduced, and it is shown that Hampel's theorem for general metrics d still holds. Since Hampel's theorem assumes the UGC property w.r.t. d, that is, convergence in probability of the empirical probability measure to the true marginal distribution w.r.t. d uniformly in the class of all admissible laws on the sample path space, this property is shown for a large class of strongly mixing laws for three different metrics d. For real-valued observations, the UGC property is established for both the Kolomogorov ϕ-metric and the Lévy ψ-metric, and for observations in a general locally compact and second countable Hausdorff space the UGC property is established for a certain metric generating the ψ-weak topology. The key is a new uniform weak LLN for strongly mixing random variables. The latter is of independent interest and relies on Rio's maximal inequality.

Keywords: ψ-weak topology; function bracket; Hampel's theorem; Kolmogorov ϕ-metric; Lévy ψ-metric; locally compact and second countable Hausdorff space; plug-in estimator; qualitative robustness; Rio's maximal inequality; strong mixing; uniform Glivenko–Cantelli theorem; uniform weak law of large numbers

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Local bilinear multiple-output quantile/depth regression

MARC HALLIN[1,2,*], ZUDI LU[3], DAVY PAINDAVEINE[1,**] and MIROSLAV ŠIMAN[4]

[1]*E.C.A.R.E.S., Université libre de Bruxelles CP114/04, 50 Avenue F.D. Roosevelt, B-1050 Brussels, Belgium. E-mail: [*]mhallin@ulb.ac.be; [**]dpaindav@ulb.ac.be; url: http://homepages.ulb.ac.be/~dpaindav*
[2]*ORFE, Princeton University, Sherrerd Hall, Princeton, NJ 08544, USA*
[3]*Mathematical Sciences & Statistical Sciences Research Institute, University of Southampton, Building 54, Highfield, Southampton SO17 1BJ, UK. E-mail: zudilu@gmail.com; url: https://sites.google.com/site/zudiluwebsite/*
[4]*Department of Stochastic Informatics, Institute of Information Theory and Automation of the ASCR, Pod Vodárenskou věží 4, CZ-182 08 Prague 8, Czech Republic. E-mail: siman@utia.cas.cz*

A new quantile regression concept, based on a directional version of Koenker and Bassett's traditional single-output one, has been introduced in [*Ann. Statist.* (2010) **38** 635–669] for multiple-output location/linear regression problems. The polyhedral contours provided by the empirical counterpart of that concept, however, cannot adapt to unknown nonlinear and/or heteroskedastic dependencies. This paper therefore introduces local constant and local linear (actually, bilinear) versions of those contours, which both allow to asymptotically recover the conditional halfspace depth contours that completely characterize the response's conditional distributions. Bahadur representation and asymptotic normality results are established. Illustrations are provided both on simulated and real data.

Keywords: conditional depth; growth chart; halfspace depth; local bilinear regression; multivariate quantile; quantile regression; regression depth

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Standard imsets for undirected and chain graphical models

TAKUYA KASHIMURA[1] and AKIMICHI TAKEMURA[1,2]

[1]*Department of Mathematical Informatics, Graduate School of Information Science and Technology, University of Tokyo, 7-3-1 Hongo, Bunkyo-ku, Tokyo, 113-8656, Japan. E-mail: takemura@stat.t.u-tokyo.ac.jp*
[2]*JST, CREST, 5 Sanbancho, Chiyoda-ku, Tokyo, 102-0075, Japan*

We derive standard imsets for undirected graphical models and chain graphical models. Standard imsets for undirected graphical models are described in terms of minimal triangulations for maximal prime subgraphs of the undirected graphs. For describing standard imsets for chain graphical models, we first define a triangulation of a chain graph. We then use the triangulation to generalize our results for the undirected graphs to chain graphs.

Keywords: conditional independence; decomposable graph; maximal prime subgraph; triangulation

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Simultaneous large deviations for the shape of Young diagrams associated with random words

CHRISTIAN HOUDRÉ* and JINYONG MA**

Georgia Institute of Technology, School of Mathematics, Atlanta, GA 30332-0160, USA.
E-mail: *houdre@math.gatech.edu; **jma@math.gatech.edu

We investigate the large deviations of the shape of the random RSK Young diagrams associated with a random word of size n whose letters are independently drawn from an alphabet of size $m = m(n)$. When the letters are drawn uniformly and when both n and m converge together to infinity, m not growing too fast with respect to n, the large deviations of the shape of the Young diagrams are shown to be the same as that of the spectrum of the traceless GUE. In the non-uniform case, a control of both highest probabilities will ensure that the length of the top row of the diagram satisfies a large deviation principle. In either case, both speeds and rate functions are identified. To complete our study, non-asymptotic concentration bounds for the length of the top row of the diagrams, that is, for the length of the longest increasing subsequence of the random word are also given for both models.

Keywords: large deviations; longest increasing subsequence; random matrices; random words; strong approximation; Young diagrams

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Convergence of the empirical spectral distribution function of Beta matrices

ZHIDONG BAI[1,*], JIANG HU[1,**], GUANGMING PAN[2] and WANG ZHOU[3]

[1]*KLASMOE and School of Mathematics & Statistics, Northeast Normal University, Changchun, P.R.C., 130024. E-mail:* [*]*baizd@nenu.edu.cn;* [**]*huj156@nenu.edu.cn*
[2]*Division of Mathematical Sciences, School of Physical and Mathematical Sciences, Nanyang Technological University, Singapore 637371. E-mail: gmpan@ntu.edu.sg*
[3]*Department of Statistics and Applied Probability, National University of Singapore, Singapore 117546. E-mail: stazw@nus.edu.sg*

Let $\mathbf{B}_n = \mathbf{S}_n(\mathbf{S}_n + \alpha_n \mathbf{T}_N)^{-1}$, where $\mathbf{S}_n$ and $\mathbf{T}_N$ are two independent sample covariance matrices with dimension p and sample sizes n and N, respectively. This is the so-called Beta matrix. In this paper, we focus on the limiting spectral distribution function and the central limit theorem of linear spectral statistics of $\mathbf{B}_n$. Especially, we do not require $\mathbf{S}_n$ or $\mathbf{T}_N$ to be invertible. Namely, we can deal with the case where $p > \max\{n, N\}$ and $p < n + N$. Therefore, our results cover many important applications which cannot be simply deduced from the corresponding results for multivariate F matrices.

Keywords: Beta matrices; CLT; LSD; multivariate statistical analysis

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Maxima of long memory stationary symmetric α-stable processes, and self-similar processes with stationary max-increments

TAKASHI OWADA[1] and GENNADY SAMORODNITSKY[2]

[1] *Faculty of Electrical Engineering, Technion, Haifa, Israel 32000. E-mail: takashiowada@ee.technion.ac.il*
[2] *School of Operations Research and Information Engineering, and Department of Statistical Science, Cornell University, Ithaca, NY 14853, USA. E-mail: gs18@cornell.edu*

We derive a functional limit theorem for the partial maxima process based on a long memory stationary α-stable process. The length of memory in the stable process is parameterized by a certain ergodic-theoretical parameter in an integral representation of the process. The limiting process is no longer a classical extremal Fréchet process. It is a self-similar process with α-Fréchet marginals, and it has stationary max-increments, a property which we introduce in this paper. The functional limit theorem is established in the space $D[0,\infty)$ equipped with the Skorohod M_1-topology; in certain special cases the topology can be strengthened to the Skorohod J_1-topology.

Keywords: conservative flow; extreme value theory; pointwise dual ergodicity; sample maxima; stable process

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The logarithmic law of random determinant

ZHIGANG BAO[1], GUANGMING PAN[2] and WANG ZHOU[3]

[1]*Department of Mathematics, Zhejiang University, Hangzhou, P.R. China.*
E-mail: maomie2007@gmail.com
[2]*School of Physical and Mathematical Sciences, Nanyang Technological University, Singapore.*
E-mail: gmpan@ntu.edu.sg
[3]*Department of Statistics and Applied Probability, National University of Singapore, Singapore.*
E-mail: stazw@nus.edu.sg

Consider the square random matrix $A_n = (a_{ij})_{n,n}$, where $\{a_{ij} := a_{ij}^{(n)}, i, j = 1, \dots, n\}$ is a collection of independent real random variables with means zero and variances one. Under the additional moment condition

$$\sup_n \max_{1 \le i, j \le n} \mathbb{E}a_{ij}^4 < \infty,$$

we prove Girko's logarithmic law of $\det A_n$ in the sense that as $n \to \infty$

$$\frac{\log|\det A_n| - (1/2)\log(n-1)!}{\sqrt{(1/2)\log n}} \xrightarrow{d} N(0, 1).$$

Keywords: CLT for martingale; logarithmic law; random determinant

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Robust estimation and inference for heavy tailed GARCH

JONATHAN B. HILL[1]

[1]*Department of Economics, University of North Carolina, Chapel Hill, NC 27599-3305, USA. E-mail: jbhill@email.unc.edu; url: www.unc.edu/~jbhill*

We develop two new estimators for a general class of stationary GARCH models with possibly heavy tailed asymmetrically distributed errors, covering processes with symmetric and asymmetric feedback like GARCH, Asymmetric GARCH, VGARCH and Quadratic GARCH. The first estimator arises from negligibly trimming QML criterion equations according to error extremes. The second imbeds negligibly transformed errors into QML score equations for a Method of Moments estimator. In this case, we exploit a sub-class of redescending transforms that includes tail-trimming and functions popular in the robust estimation literature, and we re-center the transformed errors to minimize small sample bias. The negligible transforms allow both identification of the true parameter and asymptotic normality. We present a consistent estimator of the covariance matrix that permits classic inference without knowledge of the rate of convergence. A simulation study shows both of our estimators trump existing ones for sharpness and approximate normality including QML, Log-LAD, and two types of non-Gaussian QML (Laplace and Power-Law). Finally, we apply the tail-trimmed QML estimator to financial data.

Keywords: GARCH; heavy tails; QML; robust inference; tail trimming

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Time-varying network models

HARRY CRANE

Department of Statistics & Biostatistics, Rutgers University, 110 Frelinghuysen Road, Hill Center, Room 461, Piscataway, NJ 08854, USA. E-mail: hcrane@stat.rutgers.edu

We introduce the *exchangeable rewiring process* for modeling time-varying networks. The process fulfills fundamental mathematical and statistical properties and can be easily constructed from the novel operation of *random rewiring*. We derive basic properties of the model, including consistency under subsampling, exchangeability, and the Feller property. A reversible sub-family related to the Erdős–Rényi model arises as a special case.

Keywords: Aldous–Hoover theorem; consistency under subsampling; Erdős–Rényi random graph; exchangeable random graph; graph limit; partially exchangeable array

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A criterion for invariant measures of Itô processes based on the symbol

ANITA BEHME[1] and ALEXANDER SCHNURR[2]

[1]*Institut für Mathematische Stochastik, Technische Universität Dresden, D-01062 Dresden, Germany and Technische Universität München, Center for Mathematical Sciences, Boltzmannstrasse 3, D-85748 Garching, Germany. E-mail: behme@ma.tum.de*
[2]*Department Mathematik, Universität Siegen, Walter-Flex-Str. 3, D-57072 Siegen and Technische Universität Dortmund, Lehrstuhl IV, Fakultät für Mathematik, D-44227 Dortmund, Germany. E-mail: alexander.schnurr@math.tu-dortmund.de*

An integral criterion for the existence of an invariant measure of an Itô process is developed. This new criterion is based on the probabilistic symbol of the Itô process. In contrast to the standard integral criterion for invariant measures of Markov processes based on the generator, no test functions and hence no information on the domain of the generator is needed.

Keywords: Feller process; invariant measure; Itô process; Lévy-type process; stationarity; stochastic differential equation; symbol

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Modulus of continuity of some conditionally sub-Gaussian fields, application to stable random fields

HERMINE BIERMÉ[1] and CÉLINE LACAUX[2,3,4]

[1]*MAP 5, CNRS UMR 8145, Université Paris Descartes, 45 rue des Saints-Pères, 75006 Paris, France. E-mail: Hermine.Bierme@mi.parisdescartes.fr*
[2]*Institut Élie Cartan de Lorraine, Université de Lorraine, UMR 7502, Vandœuvre-lès-Nancy, F-54506, France. E-mail: Celine.Lacaux@univ-lorraine.fr*
[3]*CNRS, Institut Élie Cartan de Lorraine, UMR 7502, Vandœuvre-lès-Nancy, F-54506, France*
[4]*INRIA, BIGS, Villers-lès-Nancy, F-54600, France*

In this paper, we study modulus of continuity and rate of convergence of series of conditionally sub-Gaussian random fields. This framework includes both classical series representations of Gaussian fields and LePage series representations of stable fields. We enlighten their anisotropic properties by using an adapted quasi-metric instead of the classical Euclidean norm. We specify our assumptions in the case of shot noise series where arrival times of a Poisson process are involved. This allows us to state unified results for harmonizable (multi)operator scaling stable random fields through their LePage series representation, as well as to study sample path properties of their multistable analogous.

Keywords: Hölder regularity; operator scaling property; stable and multistable random fields; sub-Gaussian

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Statistical estimation of a growth-fragmentation model observed on a genealogical tree

MARIE DOUMIC[1], MARC HOFFMANN[2], NATHALIE KRELL[3] and LYDIA ROBERT[4,5]

[1]*INRIA Rocquencourt, projet BANG, Domaine de Voluceau, BP 105, 781153 Rocquencourt, France and Laboratoire J.L. Lions, CNRS-UMR 7598, 4 place Jussieu, 75005 Paris, France. E-mail: marie.doumic-jauffret@inria.fr*
[2]*Université Paris-Dauphine, CNRS-UMR 7534, Place du maréchal De Lattre de Tassigny, 75775 Paris Cedex 16, France. E-mail: hoffmann@ceremade.dauphine.fr*
[3]*Université de Rennes 1, CNRS-UMR 6625, Campus de Beaulieu, 35042 Rennes Cedex, France. E-mail: nathalie.krell@univ-rennes1.fr*
[4]*INRA, UMR 1319 Micalis, F-78350 Jouy-en-Josas, France*
[5]*AgroParisTech, UMR Micalis, F-78350 Jouy-en-Josas, France. E-mail: lydia.robert@upmc.fr*

We raise the issue of estimating the division rate for a growing and dividing population modelled by a piecewise deterministic Markov branching tree. Such models have broad applications, ranging from TCP/IP window size protocol to bacterial growth. Here, the individuals split into two offsprings at a division rate $B(x)$ that depends on their size x, whereas their size grow exponentially in time, at a rate that exhibits variability. The mean empirical measure of the model satisfies a growth-fragmentation type equation, and we bridge the deterministic and probabilistic viewpoints. We then construct a nonparametric estimator of the division rate $B(x)$ based on the observation of the population over different sampling schemes of size n on the genealogical tree. Our estimator nearly achieves the rate $n^{-s/(2s+1)}$ in squared-loss error asymptotically, generalizing and improving on the rate $n^{-s/(2s+3)}$ obtained in (*SIAM J. Numer. Anal.* **50** (2012) 925–950, *Inverse Problems* **25** (2009) 1–22) through indirect observation schemes. Our method is consistently tested numerically and implemented on *Escherichia coli* data, which demonstrates its major interest for practical applications.

Keywords: cell division equation; growth-fragmentation; Markov chain on a tree; nonparametric estimation

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Interplay of insurance and financial risks in a discrete-time model with strongly regular variation

JINZHU LI[1] and QIHE TANG[2]

[1]*School of Mathematical Science and LPMC, Nankai University, Tianjin 300071, P.R. China. E-mail: lijinzhu@nankai.edu.cn*
[2]*Department of Statistics and Actuarial Science, The University of Iowa, 241 Schaeffer Hall, Iowa City, IA 52242, USA. E-mail: qihe-tang@uiowa.edu*

Consider an insurance company exposed to a stochastic economic environment that contains two kinds of risk. The first kind is the insurance risk caused by traditional insurance claims, and the second kind is the financial risk resulting from investments. Its wealth process is described in a standard discrete-time model in which, during each period, the insurance risk is quantified as a real-valued random variable X equal to the total amount of claims less premiums, and the financial risk as a positive random variable Y equal to the reciprocal of the stochastic accumulation factor. This risk model builds an efficient platform for investigating the interplay of the two kinds of risk. We focus on the ruin probability and the tail probability of the aggregate risk amount. Assuming that every convex combination of the distributions of X and Y is of strongly regular variation, we derive some precise asymptotic formulas for these probabilities with both finite and infinite time horizons, all in the form of linear combinations of the tail probabilities of X and Y. Our treatment is unified in the sense that no dominating relationship between X and Y is required.

Keywords: asymptotics; convolution equivalence; financial risk; insurance risk; ruin probabilities; (strongly) regular variation; tail probabilities

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Extinction time for a random walk in a random environment

ANNA DE MASI[1], ERRICO PRESUTTI[2], DIMITRIOS TSAGKAROGIANNIS[3] and MARIA EULALIA VARES[4]

[1]*Dipartimento di Ingegneria e Scienze dell'Informazione e Matematica, Università di L'Aquila, Via Vetoio, 1 67100 L'Aquila, Italy. E-mail: demasi@univaq.it*
[2]*Gran Sasso Science Institute, Viale Francesco Crispi,7 67100 L'Aquila, Italy. E-mail: errico.presutti@gmail.com*
[3]*Department of Mathematics, University of Sussex, Pevensey 2 Building, Falmer Campus, Brighton BN1 9QH, UK. E-mail: D.Tsagkarogiannis@sussex.ac.uk*
[4]*Instituto de Matemática, Universidade Federal do Rio de Janeiro, Av. Athos da S. Ramos 149, 21941-909, Rio de Janeiro, RJ, Brazil. E-mail: eulalia@im.ufrj.br*

We consider a random walk with death in $[-N, N]$ moving in a time dependent environment. The environment is a system of particles which describes a current flux from N to $-N$. Its evolution is influenced by the presence of the random walk and in turn it affects the jump rates of the random walk in a neighborhood of the endpoints, determining also the rate for the random walk to die. We prove an upper bound (uniform in N) for the survival probability up to time t which goes as $c\exp\{-bN^{-2}t\}$, with c and b positive constants.

Keywords: random walk in moving environment; survival probability

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Exponential rate of convergence in current reservoirs

ANNA DE MASI[1], ERRICO PRESUTTI[2], DIMITRIOS TSAGKAROGIANNIS[3] and MARIA EULALIA VARES[4]

[1]*Dipartimento di Ingegneria e Scienze dell'Informazione e Matematica, Università di L'Aquila, Via Vetoio, 1 67100 L'Aquila, Italy. E-mail: demasi@univaq.it*
[2]*Gran Sasso Science Institute, Viale Francesco Crispi, 7 67100 L'Aquila, Italy. E-mail: errico.presutti@gmail.com*
[3]*Department of Mathematics, University of Sussex, Pevensey 2 Building, Falmer Campus, Brighton BN1 9QH, UK. E-mail: D.Tsagkarogiannis@sussex.ac.uk*
[4]*Instituto de Matemática, Universidade Federal do Rio de Janeiro, Av. Athos da S. Ramos 149, 21941-909, Rio de Janeiro, RJ, Brazil. E-mail: eulalia@im.ufrj.br*

In this paper, we consider a family of interacting particle systems on $[-N, N]$ that arises as a natural model for current reservoirs and Fick's law. We study the exponential rate of convergence to the stationary measure, which we prove to be of the order N^{-2}.

Keywords: exponential convergence to the stationary measure; interacting particle systems

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On particle Gibbs sampling

NICOLAS CHOPIN[1] and SUMEETPAL S. SINGH[2]

[1]*CREST-ENSAE and HEC Paris, 3 Avenue Pierre Larousse, 92235 Malakoff, France.*
E-mail: nicolas.chopin@ensae.fr
[2]*Department of Engineering, University of Cambridge, Trumpington Street, Cambridge, CB2 1PZ, UK.*
E-mail: sss40@eng.cam.ac.uk

The particle Gibbs sampler is a Markov chain Monte Carlo (MCMC) algorithm to sample from the full posterior distribution of a state-space model. It does so by executing Gibbs sampling steps on an extended target distribution defined on the space of the auxiliary variables generated by an interacting particle system. This paper makes the following contributions to the theoretical study of this algorithm. Firstly, we present a coupling construction between two particle Gibbs updates from different starting points and we show that the coupling probability may be made arbitrarily close to one by increasing the number of particles. We obtain as a direct corollary that the particle Gibbs kernel is uniformly ergodic. Secondly, we show how the inclusion of an additional Gibbs sampling step that reselects the ancestors of the particle Gibbs' extended target distribution, which is a popular approach in practice to improve mixing, does indeed yield a theoretically more efficient algorithm as measured by the asymptotic variance. Thirdly, we extend particle Gibbs to work with lower variance resampling schemes. A detailed numerical study is provided to demonstrate the efficiency of particle Gibbs and the proposed variants.

Keywords: Feynman–Kac formulae; Gibbs sampling; particle filtering; particle Markov chain Monte Carlo; sequential Monte Carlo

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Capacity of an associative memory model on random graph architectures

MATTHIAS LÖWE[1] and FRANCK VERMET[2]

[1]*Fachbereich Mathematik und Informatik, Universität Münster, Einsteinstraße 62, 48149 Münster, Germany. E-mail: maloewe@math.uni-muenster.de*
[2]*Laboratoire de Mathématiques, UMR CNRS 6205, Université de Bretagne Occidentale, 6 avenue Victor Le Gorgeu, CS 93837, F-29238 Brest Cedex 3, France. E-mail: franck.vermet@univ-brest.fr*

We analyze the storage capacity of the Hopfield models on classes of random graphs. While such a setup has been analyzed for the case that the underlying random graph model is an Erdös–Renyi graph, other architectures, including those investigated in the recent neuroscience literature, have not been studied yet. We develop a notion of storage capacity that highlights the influence of the graph topology and give results on the storage capacity for not too irregular random graph models. The class of models investigated includes the popular power law graphs for some parameter values.

Keywords: associative memory; Hopfield model; powerlaw graphs; random graphs; random matrix; spectral theory; statistical mechanics

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Asymptotic total variation tests for copulas

JEAN-DAVID FERMANIAN[1], DRAGAN RADULOVIĆ[2]
and MARTEN WEGKAMP[3]

[1]*CREST-ENSAE, 3 av. Pierre Larousse, 92245 Malakoff Cedex, France.*
E-mail: jean-david.fermanian@ensae.fr
[2]*Department of Mathematics, Florida Atlantic University, 777 Glades Road, Boca Raton, FL 33431, USA.*
E-mail: radulovi@fau.edu
[3]*Department of Mathematics & Department of Statistical Science, Cornell University, 310 Malot Hall, Ithaca, NY 14853, USA. E-mail: marten.wegkamp@cornell.edu*

We propose a new platform of goodness-of-fit tests for copulas, based on empirical copula processes and nonparametric bootstrap counterparts. The standard Kolmogorov–Smirnov type test for copulas that takes the supremum of the empirical copula process indexed by orthants is extended by test statistics based on the empirical copula process indexed by families of L_n disjoint boxes, with L_n slowly tending to infinity. Although the underlying empirical process does not converge, the critical values of our new test statistics can be consistently estimated by nonparametric bootstrap techniques, under simple or composite null assumptions. We implemented a particular example of these tests and our simulations confirm that the power of the new procedure is oftentimes higher than the power of the standard Kolmogorov–Smirnov or the Cramér–von Mises tests for copulas.

Keywords: bootstrap; copula; empirical copula process; goodness-of-fit test; weak convergence

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